The torque is :

54a) $$TRQ_{INST.} = Dp\int_{X_i}^{C_r}\int_{-[C_r^2 - x^2]^{1/2}}^{[C_r^2 - x^2]^{1/2}} \big[R + x_m SIN\,\mu + y_n SIN\,G + P_r SIN(G\text{-}A(59) + \sigma)\big]\, dy\, dx,$$

Using $A = R + P_r SIN(G - A(59) + \sigma)$, $B = SIN\,G$, and $C = SIN\,\mu$.

54b) $$TRQ_{INST.} = dP\int_{X_1}^{C_r}\;{}^{*}\,Ay + By^2/2 + Cyx\,dx\Big|_{-[C_r^2 - x^2]^{1/2}}^{[C_r^2 - x^2]^{1/2}} = dP\Big[\int_{X_i}^{C_r} 2A\,[C_r^2 - x^2]^{1/2}\,dx + \int_{X_i}^{C_r} 2\,x\,C[C_r^2 - x^2]^{1/2}\,dx\Big].$$

Using the substitutions $x = C_r COS\,\sigma$, and $dx = - C_r SIN\,\sigma\, d\sigma$ in the first integration the right for its solution:

54b1) $$-2\,A\,dP\,C_r^2\int_{ACOS(X_i/C_r)}^{0}[1 - COS^2\sigma]^{1/2}\,SIN\,\sigma\,d\sigma = dP\,\big[R + P_r SIN(G - A(59) + \sigma)\big]\big[C_r^2 ACS(X_i/C_r) - X_i[C_r^2 - X_i^2]^{1/2}\big].$$

{Fn MRAD} {Fn IVC } {Fn SQA}

Using the same substitutions, the second integral solution becomes:

54b2) $$dP\int_{X_i}^{C_r} 2\,x\,C\,[C_r^2 - x^2]^{1/2}\,dx = -2/3\,dP\,C\,{}^{*}[C_r^2 - x^2]^{3/2}\Big|_{X_i}^{C_r} = 2/3\,dP\,SIN\,\mu\,[C_r^2 - X_i^2]^{3/2}.$$

Combining the solutions of 54b1 and 54b2, the instant torque equation is:

54b3) $$TRQ_{INST.} = dP\,\big[R + P_r SIN(G - A(59) + \sigma)\big]\big(C_r^2\,ACS(X_i/C_r) - X_i[C_r^2 - X_i^2]^{1/2}\big) + 2/3\,dP\,SIN\,\mu\,[C_r^2 - X_i^2]^{3/2}.$$

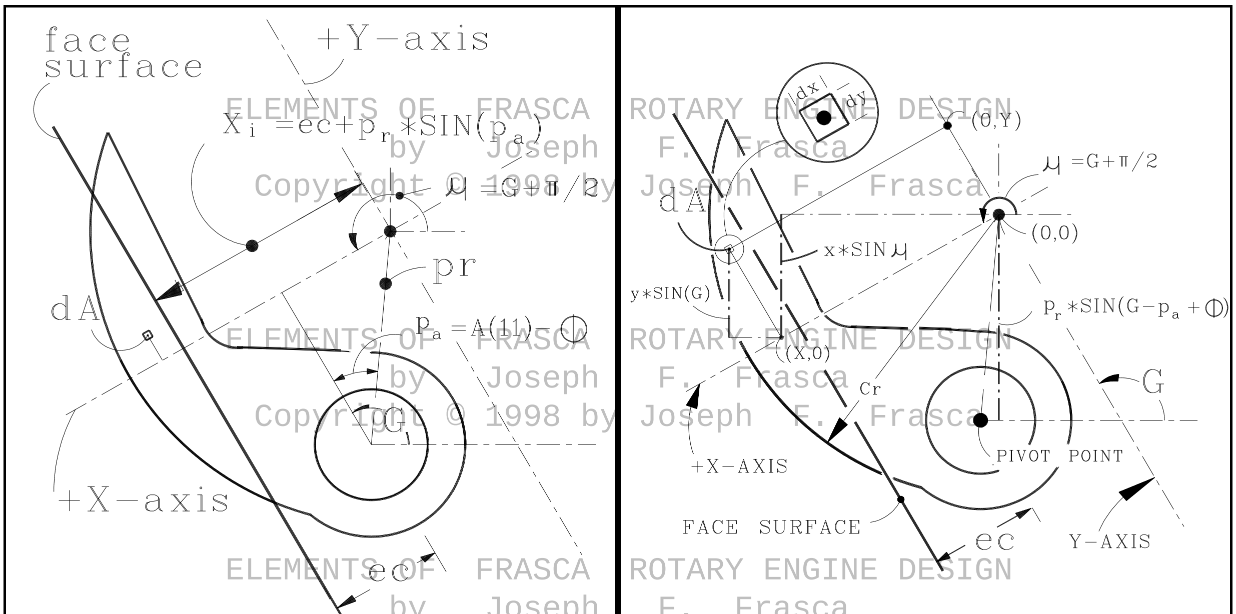

FIGURE 54a PARTITION COORDINATE ELEMENTS. FIGURE 54b MOMENT ARM COMPONENTS.

55 CIR ENGINE MINIMUM AND MAXIMUM VVC NET VOLUME.

Rotation through a differential arc, $d\theta$, of a differential area dA at moment arm distance, M_A, from the rotor axis, generates a differential volume dV; i.e. $dV = dx\,dy\,M_A\,d\theta$. The volume integral is:

55) $$VOL_{INST.} = \int_{\theta_1}^{\theta_2}\int_{X_i}^{C_r}\int_{-[C_r^2 - x^2]^{1/2}}^{[C_r^2 - x^2]^{1/2}}\big[R + x_m N\mu + y_n SIN(G) + P_r SIN(G - A(59) + \sigma)\big]\,dy\,dx\,d\theta.$$

This becomes:

55a) $$VOL_{INST.} = \int_{\theta_1}^{\theta_2}\big[R + P_r SIN(G - A(59) + \sigma)\big]\big(C_r^2\,ACS(X_i/C_r) - X_i\,[C_r^2 - X_i^2]^{1/2}\big) + 2/3\,SIN\,\mu\,[C_r^2 - X_i^2]^{3/2}\,d\theta.$$

In constant volume regions of the annular cavity the above integral's solution is:

55b) $$VOL_{constant} = [R + P_r SIN(G - A(59) + \sigma)](C_r^2 ACS(X_i/C_r) - X_i[C_r^2 - X_i^2]^{1/2})(\theta_2 - \theta_1) + 2/3\,SIN\,\mu\,[C_r^2 - X_i^2]^{3/2}(\theta_2 - \theta_1).$$

Where $\sigma = 0$, $X_i = X_{max} = P_r\,SIN(A(59)) + ec$, and a VVC has minimum volume:

55c) $$VOL_{min} = [R + P_r SIN(G - A(59))](C_r^2 ACS(X_{max}/C_r) - X_{max}[C_r^2 - X_{max}^2]^{1/2})(\theta_2 - \theta_1) + 2/3\,SIN\,\mu\,[C_r^2 - X_{max}^2]^{3/2}(\theta_2 - \theta_1).$$

Where $\sigma = \alpha_2$ and $X_{min} = P_r SIN(A(59) - \alpha_2) + e_c$, the VVC has maximum volume:

55d) $$VOL_{max} = [R + P_r\,SIN(G - A(59) + \alpha_2)](C_r^2 ACS(X_{min}/C_r) - X_{min}\,[C_r^2 - {}_{min}^2]^{1/2})(\theta_2 - \theta_1) + 2/3\,SIN\,\mu[C_r^2 - X_{min}^2]^{3/2}(\theta_2 - \theta_1).$$

The net maximum VVC volume is:

55$_e$) $_{VVC}VOL_{NET\ MAX} = VOL_{max} - PARVOL_{AT\ \phi=\epsilon=\alpha_1+\alpha_2}$,

and the net minimum VVC volume is:

55$_{e1}$) $_{VVC}VOL_{NET\ MIN} = VOL_{min} - PARVOL_{AT\ \phi=\alpha 1}$.

The program increases by very small increments the value of α_1 and dependent constants, A(59), σ, X_m, α_2, VOL_{min} and $PARVOL_{AT\ \phi=\alpha 1}$ until the value of: $[_{VVC}VOL_{NET\ MAX}/_{VVC}VOL_{NET\ MIN}]$, is adequately close to the volume ratio desired. Solution of equation 55 in the volume varying portions of the annular cavity will be discussed later.

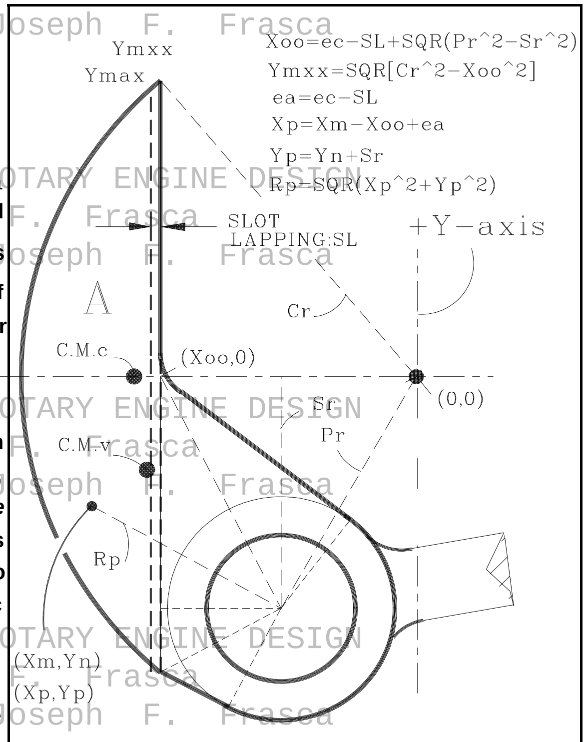

FIGURE 56 PARTITION SEGMENTS.

56 THE CIRCLE SEGMENT INERTIAL MOMENT.

Except that in partitions with tapered circular section whose thickness factor is dependent on the distance, D_{PR}, between a point in the PCS, (x_p, y_q) and the pivot point, *see equation 56 below,* and the derivation of the circular section's moment of inertia about the pivot point, the methods used to find the previous engines' partition inertial moment hold in topic engine designs. Refer to figure 56.

56) $D_{PR} = [(x_p - X_{oo} + eca)^2 + (S_r + y_q)^2]^{1/2}$.

Returning to the constant thickness partition, we'll find the moment of inertia of the circular section about the pivot point using the parallel axis theorem. This method may also be used with tapered partitions. The moment of inertia in polar coordinates of the circular section about an axis through the PCS origin and perpendicular to the partition's axial plane follows from:

56$_a$) $$_{CIR}I_{(0,0)} = T_K\int_{X_{oo}}^{C_r}\int_{-(C_r^2-x^2)^{1/2}}^{(C_r^2-x^2)^{1/2}}(x^2+y^2)\,dx\,dy = 2T_K\int_{0}^{ACOS(X_{oo}/C_r)}\int_{X_{oo}SEC\,\epsilon}^{C_r} r^2\,r\,d\epsilon\,dr$$
$$= T_K\left[1/2\,C_r^4 ACS(X_{oo}/C_r) - X_{oo}/6\,(C_r^2-X_{oo}^2)^{3/2} - X_{oo}^3/2\,(C_r^2-X_{oo}^2)^{1/2}\right].$$

We next find the circle section center of mass C.M. in the PCS. As the partition thickness is constant in our current discussion the C.M. will be on the X axis; i.e. $Y_{C.M.} = 0$. The center of mass on the X - axis is:

56$_b$) $$X_{C.M.} = \frac{\int\int rCOS(\epsilon)\,r\,d\epsilon\,dr}{\int\int r\,d\epsilon\,dr} = \frac{2/3\,(C_r^2 - X_{oo}^2)^{3/2}}{C_r^2/2\,ACS(X_{oo}/C_r) - X_{oo}/2\,(C_r^2 - X_{oo}^2)^{1/2}}.$$

The moment of inertia at the center of mass, $_{CIR}I_{C.M.}$ follows from:

56$_c$) $$_{CIR}I_{C.M.} = {}_{CIR}I_{(0,0)} - T_k\left[C_r^2/2ACS(X_{oo}/C_r) - X_{oo}/2\,(C_r^2 - X_{oo}^2)^{1/2}\right](X_{C.M.}^2 + Y_{C.M.}^2).\ [Y_{C.M.} = 0]$$

The Y distance between the center of mass of the topic circle section and the pivot is the swing radius, S_r; i.e. $Y_{P\text{-}C.M.} = S_r$, and the x distance between the center of mass and the pivot is given by: $X_{P\text{-}C.M.} = X_{C.M.} - [P_r^2 - S_r^2]^{1/2}$. Using the parallel axis theorem again, the partition's circle segment moment of inertial at the pivot is:

56$_d$) $$_{CIR}I_{PIVOT} = {}_{CIR}I_{C.M.} + T_k\left[C_r^2/2\,ACS(X_{oo}/C_r) - X_{oo}/2\,(C_r^2 - X_{oo}^2)^{1/2}\right](X_{P\text{-}C.M.}^2 + Y_{P\text{-}C.M.}^2).$$

57 THE ANNULAR CAVITY VOLUME BY INTEGRAL.

As noted in section 55, the volume between two cavity angles θ_2 and θ_1 is given by integral,

57) $$VOL_{INST.} = \int_{\theta_1}^{\theta_2}\int_{X_x}^{C_r}\int_{-[C_r^2-x^2]^{1/2}}^{[C_r^2-x^2]^{1/2}}[R + x_m SIN\mu + y_n SIN(G) + P_r SIN(G - A(59) + \sigma)]\,dy\,dx\,d\theta,$$

57$_a$) $$= \int_{\theta_1}^{\theta_2}[R + P_r SIN(G - A(59) + \sigma)]\left(C_r^2\,ACS(X_i/C_r) - X_i[C_r^2 - X_i^2]^{1/2}\right) + 2/3\,SIN\mu\,[C_r^2 - X_i^2]^{3/2}\,d\theta.$$

For simplicity we modify the integral with $\zeta = u(\theta - \beta)$ and $d\zeta = u\,d\theta$. (ζ is zeta)

57$_c$) $$VOL_{INST.} = 1/u\int_{\zeta_1 = u(\theta_1-\beta)}^{\zeta_2 = u(\theta_2-\beta)}[R + P_r SIN(G - A(59) + \sigma)]\left(C_r^2\,ACS(X_i/C_r) - X_i[C_r^2 - X_i^2]^{1/2}\right) + 2/3\,SIN\mu[C_r^2 - X_i^2]^{3/2}\,d\zeta.$$

This modifies the interval of integration from the rotor coordinate system (RCS) system with independent variable θ to the undulation coordinate system (UCS) with independent variable ζ (CA AN in the previously discussed engine designs) with intervals of integration from 0 to π and π to 2π.

To solve the above integral we first divide the interval of integration, 0 to π, into 24 sequential and equal intervals: [0, π/24], [π/24, π/12], [π/12, π/8], [π/8, π/6] ,. [23π/24,π], and we use the Taylor series expansion (see appendix B) to generate polynomial expansion equivalents for the above integrand's terms in each of the twenty four intervals; i.e.

$$y = g(x) \approx f(\mu) + f^1(\mu)(\zeta - \mu) + f^2(\mu)/2!\,(\zeta - \mu)^2 + f^3(\mu)/3!\,(\zeta - \mu)^3 + .. + f^n(\mu)/n!\,(\zeta - \mu)^n.$$

The notation used here is similar to that used in the programming. The near point μ used in each interval's Taylor expansion in the form [interval, μ], are: [1, 0], [2, π/24], [3, π/12], [4, π/8], [5, π/6], [6, 5π/25], ., [24, 23π/24].

58 THE POWER EXPANSIONS FOR σ AND POWERS OF THE σ EXPANSIONS.

We begin with the equation for partition angular displacement σ:

58) $\sigma = \alpha_2/2\,[1 - COS\,u(\theta - ß)] \equiv \alpha_2/2\,[1 - COS\,\zeta] \quad 0 \le \zeta \le \pi,$

and generate a 16 term polynomial for each of the 24 intervals of the form:

58$_a$) $\sigma \approx f(\mu) + f^1(\mu)(\zeta - \mu) + f^2(\mu)/2!\,(\zeta - \mu)^2 + f^3(\mu)/3!\,(\zeta - \mu)^3 .. f^{15}(\mu)/15!\,(\zeta - \mu)^{15}.$

The coefficients of the Taylor expansion terms for σ, in each interval phc, are the constants O(phc,n) where phc is the interval number and n is the term number of the expansion in that interval; i.e.

58$_b$) $\sigma = O(phc,1) + O(phc,2)(\zeta - \mu) + (phc,3)(\zeta - \mu)^2 + O(phc,4)(\zeta - \mu)^3 + .. + O(phc,16)(\zeta - \mu)^{15}.$

Repeated differentiation of Equation 58 is used to acquire the coefficients for the σ Tayor expansion in 58b; i.e.

$O(phc,1) = \alpha_2/2\,[1 - COS\,\mu]$, $O(phc,2) = \alpha_2/2\,SIN\,\mu$, $O(phc,3) = \alpha_2/2\,[COS\,\mu]/2$, $O(phc,4) = -\alpha_2/2\,[SIN\,\mu]/6$,

$O(phc,5) = -\alpha_2/2\,[COS\,\mu]/24$, $O(phc,6) = \alpha_2/2\,[SIN\,\mu]/120$, $O(phc,7) = \alpha_2/2\,[COS\,\mu]/720$, $O(phc,8) = -\alpha_2/2\,[SIN\,\mu]/5040$,

$O(phc,9) = -\alpha_2/2\,[COS\,\mu]/40320$, $O(phc,10) = \alpha_2/2\,[SIN\,\mu]/362880$, $O(phc,11) = \alpha_2/2\,[COS\,\mu]/3628800$,

$O(phc,12) = -\alpha_2/2\,[SIN\,\mu]/39916800$, $O(phc,13) = -\alpha_2/2\,[COS\,\mu]/479001600$, $O(phc,14) = \alpha_2/2\,[SIN\,\mu]/6227020800$,

$O(phc,15) = \alpha_2/2\,[COS\,\mu]/87178291200$, and $O(phc,16) = -\alpha_2/2\,[SIN\,\mu]/1307674368000$.

Once the 24 expansion for σ in the 24 intervals are acquired, all required expansions for the remaining terms of the volume integrand are acquired completely in one interval before they're acquired for the next sequential interval. The Taylor expansion for integrand elements: $X_i = ec + P_r SIN(A(59) - \sigma)$, $[C_r^2 - X_i^2]$ and $[R + P_r SIN(A(59) - P_a + \sigma)]$ all require expansions of σ and powers of σ.

The program uses a subroutine which rapidly forms the product of (the coefficients) two multiple term polynomials. In all said expansions if the term is 0 or doesn't exist the coefficient is retained and set to 0.

For each interval, phc, the powers of σ are expressed with coefficients E(n,m), where n is the power of σ and m is the expansion's coefficient number; i.e.

$$\sigma = E(1,1) + E(1,2)(\zeta - \mu) + E(1,3)(\zeta - \mu)^2 + E(1,4)(\zeta - \mu)^3 + .. + E(1,39)(\zeta - \mu)^{38} + E(1,40)(\zeta - \mu)^{39}$$

$$\sigma^2 = E(2,1) + E(2,2)(\zeta - \mu) + E(2,3)(\zeta - \mu)^2 + E(2,4)(\zeta - \mu)^3 + .. + E(2,39)(\zeta - \mu)^{38} + E(2,40)(\zeta - \mu)^{39}$$

$$\sigma^3 = E(3,1) + E(3,2)(\zeta - \mu) + E(3,3)(\zeta - \mu)^2 + E(3,4)(\zeta - \mu)^3 + .. + E(3,39)(\zeta - \mu)^{38} + E(3,40)(\zeta - \mu)^{39}$$

$$\vdots$$

$$\sigma^{14} = E(14,1) + E(14,2)(\zeta - \mu) + E(14,3)(\zeta - \mu)^2 + E(14,4)(\zeta - \mu)^3 + \ldots + E(14,40)(\zeta - \mu)^{39}$$

$$\sigma^{15} = E(15,1) + E(15,2)(\zeta - \mu) + E(15,3)(\zeta - \mu)^2 + E(15,4)(\zeta - \mu)^3 + \ldots + E(15,40)(\zeta - \mu)^{39}.$$

E.g. In interval three:

$$\sigma = E(1,1) + E(1,2)(\zeta - \mu) + E(1,3)(\zeta - \mu)^2 + E(1,4)(\zeta - \mu)^3 + .. + E(1,39)(\zeta - \mu)^{38} + E(1,40)(\zeta - \mu)^{39}$$

$$\sigma = O(3,1) + O(3,2)(\zeta - \mu) + O(3,3)(\zeta - \mu)^2 + O(3,4)(\zeta - \mu)^3 + .. + O(3,16)(\zeta - \mu)^{15} + 0 \ldots + O(3,40)(\zeta - \mu)^{39}.$$

I.e. E(1,1)= O(3,1), E(1,2)= O(3,2), E(1,3)= O(3,3) ...

59 EXPANSIONS $X_i = e_c + P_r SIN(P_a - \sigma)$, $[R + P_r SIN(A(22) - P_a + \sigma)]$ & $[C_r^2 - X_i^2]$.

We'll use the Maclaurin expansions to approximate *SIN* σ and *COS* σ in each interval; i.e.

59) $COS\,\sigma = 1 - \sigma^2/2 + \sigma^4/24 - \sigma^6/720 + \sigma^8/40320 - \sigma^{10}/3228800 + \sigma^{12}/479001600 - \sigma^{14}/87178291200$

59$_a$) $SIN\,\sigma = \sigma - \sigma^3/6 + \sigma^5/120 - \sigma^7/5040 + \sigma^9/362880 - \sigma^{11}/39916800 + \sigma^{13}/6227020800 - \sigma^{15}/130767436800$

THE POWER EXPANSION FOR $X_i = ec + P_r SIN(P_a - \sigma)$.

Converting X_i, $X_i = e_c + P_r SIN(P_a - \sigma) = e_c + P_r SIN(P_a) COS\,\sigma - P_r SIN\,\sigma\, COS(P_a).$

59_b) $X_i = e_c + P_r SIN P_a [1 - \sigma^2/2 + \sigma^4/24 - \sigma^6/720 + \sigma^8/40320]$

$+ P_r SIN P_a [-\sigma^{10}/3228800 + \sigma^{12}/479001600 - \sigma^{14}/87178291200]$

$- P_r COS P_a [\sigma - \sigma^3/6 + \sigma^5/120 - \sigma^7/5040 + \sigma^9/362880 - \sigma^{11}/39916800]$

$- P_r COS P_a [\sigma^{13}/6227020800 - \sigma^{15}/130767436800]$.

The collection of terms in 59c below represents the 40 term expansions for consecutively increasing powers of σ used to find X_i in any of the 24 intervals.

For clutter control: $A = -P_r COS(P_a)$, and $B = P_r SIN(P_a)$.

59_c)

$$\begin{aligned}
X_i = e_c + B + \; & A\,[E(1,1) + E(1,2)(\zeta-\mu) + E(1,3)(\zeta-\mu)^2 + \dots + E(1,40)(\zeta-\mu)^{39}] \\
& + B/2\,[E(2,1) + E(2,2)(\zeta-\mu) + E(2,3)(\zeta-\mu)^2 + \dots + E(2,40)(\zeta-\mu)^{39}] \\
& - A/6\,[E(3,1) + E(3,2)(\zeta-\mu) + E(3,3)(\zeta-\mu)^2 + \dots + E(3,40)(\zeta-\mu)^{39}] \\
& + B/24\,[E(4,1) + E(4,2)(\zeta-\mu) + E(4,3)(\zeta-\mu)^2 + \dots + E(4,40)(\zeta-\mu)^{39}] \\
& \quad \vdots \\
& - B/87178291200\,[E(14,1) + E(14,2)(\zeta-\mu) + E(14,3)(\zeta-\mu)^2 + \dots + E(14,40)(\zeta-\mu)^{39}] \\
& - A/130767436800\,[E(15,1) + E(15,2)(\zeta-\mu) + E(15,3)(\zeta-\mu)^2 + \dots + E(15,40)(\zeta-\mu)^{39}] \\
\hline
& U(phc,1) + U(phc,2)(\zeta-\mu) + U(phc,3)(\zeta-\mu)^2 + \dots + U(phc,40)(\zeta-\mu)^{39}
\end{aligned}$$

The program multiplies the above bracketed expansions by their cofactors and then sums the resultant coefficients of like powers of $(\zeta - \mu)$ for the expansion equation for X_i in each interval phc, and where U(phc,1) = U(phc,1) + ec + B.

59_d) $_{phc}X_i = U(phc,1) + U(phc,2)(\zeta-\mu) + \dots\dots + U(phc,39)(\zeta-\mu)^{38} + U(phc,40)(\zeta-\mu)^{39}$.

THE POWER EXPANSIONS FOR X_i^2.

In each interval phc the expansion for X_i is multiplied by itself for the expansion for X_i^2 in the interval:

59_e) $_{phc}X_i^2 = I(phc,1) + I(phc,2)(\zeta-\mu) + \dots + I(phc,39)(\zeta-\mu)^{38} + I(phc,40)(\zeta-\mu)^{39}$.

THE POWER EXPANSIONS FOR $[C_r^2 - X_i^2]$.

The power expansions for $[C_r^2 - X_i^2]$ are acquired by multiplying the coefficients of the X_i^2 expansion in each interval by (- 1) then adding C_r^2 to the first coefficient in each expansion. The resulting expansions are of the form:

59_f) $_{phc}[C_r^2 - X_i^2] = R(phc,1) + R(phc,2)(\zeta-\mu) + \dots + R(phc,39)(\zeta-\mu)^{38} + R(phc,40)(\zeta-\mu)^{39}$.

60 THE POWER EXPANSIONS FOR $[C_r^2 - X_i^2]^{1/2}$ AND ACOS (X_i/C_r). *[I.E. $COS^{-1}(X_i/C_r)$]*

We now use X_i as our independent variable in the Taylor expansion representations of $[C_r^2 - X_i^2]^{1/2}$ and ARC(X_i/C_r); i.e.

$$f(X_i) = f(X_z) + f^1(X_z)(X_i - X_z) + 1/2!f^2(X_z)(X_i - X_z)^2 + \dots + 1/n!f^n(X_z)(X_i - X_z)^n.$$

Noting in section 58 that the first term in each of the expansions for the powers of σ in each of the 24 intervals, phc, is a power of $\sigma_\mu = \alpha_2/2\,[1 - COS\,\mu] = O(phc,1)$; i.e.

$$E(1,1) = \sigma_\mu,\quad E(2,1) = \sigma_\mu^2,\quad E(3,1) = \sigma_\mu^3,\quad E(4,1) = \sigma_\mu^4, \dots,\quad E(14,1) = \sigma_\mu^{14},\quad E(15,1) = \sigma_\mu^{15}.$$

From 59c the first term, U(phc,1) in each of the power expansions of σ in $(\zeta - \mu)$ for X_i in interval phc is ec added to the sum of terms: $B + A\,E(1,1) - B/2\,E(2,1) - A/6\,E(3,1) + B/24\,E(4,1) + \dots + \dots$

Collecting terms with common coefficients:

$$B[1 - 1/2\,\sigma_\mu^2 + 1/24\,\sigma_\mu^4 - 1/720\,\sigma_\mu^6 + .] \equiv B\,COS\,\sigma_\mu, \text{ and } A[\sigma_\mu - 1/6\,\sigma_\mu^3 + 1/120\,\sigma_\mu^5 + .] \equiv A\,SIN\,\sigma_\mu.$$

The term U(phc,1), the first term of the expansion for X_i in each interval phc, has the form: $e_c + B\,COS\,\sigma_\mu + A\,SIN\,\sigma_\mu$, **but** $B = P_r SIN(P_a)$ **and** $A = -P_r COS(P_a)$, **and so**

$$U(phc,1) = e_c + P_r SIN(P_a) COS\,\sigma_\mu - P_r COS(P_a) SIN\,\sigma_\mu = ec + P_r SIN(P_a - \sigma_\mu) = X_z\,;$$

therefore, X_z is the first value of X_i in each interval phc; i.e. X_z = U(phc,1) is the near point of the X_i expansion in interval phc, and any term of the form $[X_i - X_z]$ is accurately represented in any interval by the expansion in $(\zeta - \mu)$ for X_i with the first expansion coefficient set to 0; e.g. In interval phc = 3,

$$[X_i - X_z] = U(3,2)(\zeta-\mu) + U(3,3)(\zeta-\mu)^2 + \dots + U(3,39)(\zeta-\mu)^{38} + U(3,40)(\zeta-\mu)^{39}.$$

The expansions representing powers of $[X_i - X_z]$ in any interval, phc, have the general form:

60) $_{phc}[X_i - X_z]^n = M_A(n,1) + M_A(n,2)(\zeta-\mu) + \dots\dots + M_A(n,39)(\zeta-\mu)^{38} + M_A(n,40)(\zeta-\mu)^{39}$,

60_1) $_{phc}[X_i - X_z] = M_A(1,1) + M_A(1,2)(\zeta-\mu) + \dots\dots + M_A(1,39)(\zeta-\mu)^{38} + M_A(1,40)(\zeta-\mu)^{39}$,

$_{phc}[X_i - X_z]^2 = M_A(2,1) + M_A(2,2)(\zeta-\mu) + \dots\dots + M_A(2,39)(\zeta-\mu)^{38} + M_A(2,40)(\zeta-\mu)^{39}$,

: : : :

$_{phc}[X_i - X_z]^{11} = M_A(11,1) + M_A(11,2)(\zeta-\mu) + \dots\dots + M_A(11,39)(\zeta-\mu)^{38} + M_A(11,40)(\zeta-\mu)^{39}$,

Where $M_A(n,1) = 0$, and n = 1,2,3.........11.

Power expansions in $[X_i - X_z]$ are generated for each interval using the Taylor series.

60a) $f(X_i) = f(X_Z) + f^1(X_Z)(X_i - X_Z) + 1/2!f^2(X_Z)(X_i - X_Z)^2 + \ldots + 1/n!f^n(X_Z)(X_i - X_Z)^n$.

Using expansion 60a to express the value of $[C_r - X_i]^{1/2}$ in any interval phc, and the constant X_z in all intervals phc is:

60b) $X_z = e_c + P_r SIN(P_a - \alpha_2/2\ [1 - COS((phc - 1)\pi/24)])$.

THE POWER EXPANSIONS FOR $[C_r - X_i]^{1/2}$.

The 60a expansions and the 60b relationship are used to evaluate $[C_r - X_i]^{1/2}$ and X_Z, respectively. The coefficients for the $[C_r - X_i]^{1/2}$ Taylor expansion in $(X_i - X_Z)$ are:

$N_X(0) = f(X_Z) = [C_r - X_z]^{1/2}$, $N_X(6) = f^6(X_Z) = -945/46080\ [C_r - X_z]^{-11/2}$,

$N_X(1) = f^1(X_Z) = -1/2\ [C_r - X_z]^{-1/2}$, $N_X(7) = f^7(X_Z) = -10395/645120\ [C_r - X_z]^{-13/2}$,

$N_X(2) = f^2(X_Z) = -1/8\ [C_r - X_z]^{-3/2}$, $N_X(8) = f^8(X_Z) = -135135/10321920\ [C_r - X_z]^{-15/2}$,

$N_X(3) = f^3(X_Z) = -3/48\ [C_r - X_z]^{-5/2}$, $N_X(9) = f^9(X_Z) = -2027025/185794560\ [C_r - X_z]^{-17/2}$,

$N_X(4) = f^4(X_Z) = -5/128\ [C_r - X_z]^{-7/2}$, $N_X(10) = f^{10}(X_Z) = -34459425/3715891200\ [C_r - X_z]^{-19/2}$,

$N_X(5) = f^5(X_Z) = -105/3840\ [C_r - X_z]^{-9/2}$, $N_X(11) = f^{11}(X_Z) = -654729075/81749606400\ [C_r - X_z]^{-21/2}$.

The Taylor expansion for $[C_r - X_i]^{1/2}$ is now:

$$[C_r - X_i]^{1/2} = N_X(0) + N_X(1)(X_i - X_Z) + N_X(2)(X_i - X_Z)^2 + N_X(3)(X_i - X_Z)^3 + N_X(4)(X_i - X_Z)^4 + N_X(5)(X_i - X_Z)^5$$
$$+ N_X(6)(X_i - X_Z)^6 + N_X(7)(X_i - X_Z)^7 + N_X(8)(X_i - X_Z)^8 + N_X(9)(X_i - X_Z)^9 + N_X(10)(X_i - X_Z)^{10} + N_X(11)(X_i - X_Z)^{11}.$$

Applying the above coefficients to the power expansions of $(X_i - X_Z)$ in an interval phc for the equivalent expansions of $[C_r - X_i]^{1/2}$ in $(\zeta - \mu)$, in said interval we have:

$$N_X(0) + N_X(1)(X_i - X_Z) = N_X(1)[\,M_A(1,1) + M_A(1,2)(\zeta - \mu) + \ldots + M_A(1,39)(\zeta - \mu)^{38} + M_A(1,40)(\zeta - \mu)^{39}\,]$$
$$N_X(2)(X_i - X_Z)^2 = N_X(2)[\,M_A(2,1) + M_A(2,2)(\zeta - \mu) + \ldots + M_A(2,39)(\zeta - \mu)^{38} + M_A(2,40)(\zeta - \mu)^{39}\,]$$
$$\vdots$$
$$N_X(11)(X_i - X_Z)^{11} = N_X(11)[\,M_A(11,1) + M_A(11,2)(\zeta - \mu) + \ldots + M_A(11,39)(\zeta - \mu)^{38} + M_A(11,40)(\zeta - \mu)^{39}\,]$$

60c) ${}_{phc}[C_r - X_i]^{1/2} = P(phc,1) + P(phc,2)(\zeta - \mu) + \ldots\ldots\ldots + P(phc,39)(\zeta - \mu)^{38} + P(phc,40)(\zeta - \mu)^{39}$.

The P(phc,1) term above is the sum: $P(phc,1) \equiv [\,N_X(1)M_A(1,1) + N_X(2)M_A(2,1)\ \ldots\ N_X(11)M_A(11,1)\,] + N_X(0) \equiv N_X(0)$.

THE POWER EXPANSIONS FOR $[C_r + X_i]^{1/2}$.

The coefficients of the $(X_i - X_Z)$ power expansions for the $[C_r + X_i]^{1/2}$ Taylor expansions are:

$N_Y(0) = f(X_Z) = [C_r + X_z]^{1/2}$, $N_Y(1) = f^1(X_Z) = 1/2\ [C_r + X_z]^{-1/2}$,

$N_Y(2) = f^2(X_Z) = -1/8\ [C_r + X_z]^{-3/2}$ $N_Y(3) = f^3(X_Z) = 3/48\ [C_r + X_z]^{-5/2}$,

$N_Y(4) = f^4(X_Z) = -5/128\ [C_r + X_z]^{-7/2}$, $N_Y(5) = f^5(X_Z) = 105/3840\ [C_r + X_z]^{-9/2}$

$N_Y(6) = f^6(X_Z) = -945/46080[C_r + X_z]^{-11/2}$ $N_Y(7) = f^7(X_Z) = 10395/645120\ [C_r + X_z]^{-13/2}$,

$N_Y(8) = f^8(X_Z) = -135135/10321920\ [C_r + X_z]^{-15/2}$, $N_Y(9) = f^9(X_Z) = 2027025/185794560\ [C_r + X_z]^{-17/2}$

$N_Y(10) = f^{10}(X_Z) = -34459425/3715891200[C_r + X_z]^{-19/2}$ $N_Y(11) = f^{11}(X_Z) = 654729075/81749606400[C_r + X_z]^{-21/2}$

The Taylor expansion for $[C_r + X_i]^{1/2}$ is:

$$N_Y(0) + N_Y(1)(X_i - X_Z) + N_Y(2)(X_i - X_Z)^2 + N_Y(3)(X_i - X_Z)^3 + N_Y(4)(X_i - X_Z)^4 + N_Y(5)(X_i - X_Z)^5 + N_Y(6)(X_i - X_Z)^6$$
$$+ N_Y(7)(X_i - X_Z)^7 + N_Y(8)(X_i - X_Z)^8 + N_Y(9)(X_i - X_Z)^9 + N_Y(10)(X_i - X_Z)^{10} + N_Y(11)(X_i - X_Z)^{11}.$$

Applying the above coefficients to the power expansions of $(X_i - X_Z)$ in an interval phc for the equivalent expansions of $[C_r + X_i]^{1/2}$ in $(\zeta - \mu)$, in said interval we have:

$$N_Y(0) + N_Y(1)(X_i - X_Z) = N_Y(1)[M_A(1,1) + M_A(1,2)(\zeta - \mu) + \ldots + M_A(1,39)(\zeta - \mu)^{38} + M_A(1,40)(\zeta - \mu)^{39}]$$
$$N_Y(2)(X_i - X_Z)^2 = N_Y(2)[M_A(2,1) + M_A(2,2)(\zeta - \mu) + \ldots + M_A(2,39)(\zeta - \mu)^{38} + M_A(2,40)(\zeta - \mu)^{39}]$$
$$\vdots$$
$$N_Y(11)(X_i - X_Z)^{11} = N_Y(11)[M_A(11,1) + M_A(11,2)(\zeta - \mu) + \ldots + M_A(11,39)(\zeta - \mu)^{38} + M_A(11,40)(\zeta - \mu)^{39}]$$

60d) phc[Cr - Xi]1/2 = $F(phc,1) + F(phc,2)(\zeta - \mu) + \ldots + F(phc,39)(\zeta - \mu)^{38} + F(phc,40)(\zeta - \mu)^{39}$.

Where $F(phc,1) = F(phc,1) + N_Y(0)$

POWER EXPANSION FOR ACOS (X_i/C_r) *[I.E. $COS^{-1}(X_i/C_r)$]*

Utilizing *ACOS* (X_i/C_r) = *ATN* $[(C_r^2 - X_i^2)^{1/2}/X_i]$, the cCoefficients of the Taylor expansion in $(X_i - X_z)$ for *ACOS* (X_i/C_r) are:

$N_Z(0) = ATN\ [(C_r^2 - X_Z^2)^{1/2}/X_Z]$,

$N_Z(1) = -[C_r^2 - X_Z^2]^{-1/2}$,

$N_Z(2) = -1/2\ X_Z[C_r^2 - X_Z^2]^{-3/2}$,

$N_Z(3) = -1/2\ X_Z^2[C_r^2 - X_Z^2]^{-5/2} - 1/6\ [C_r^2 - X_Z^2]^{-3/2}$,

$N_Z(4) = -5/8\ X_Z^3[C_r^2 - X_Z^2]^{-7/2} - 3/8\ X_Z[C_r^2 - X_Z^2]^{-5/2}$,

$N_Z(5) = -105/120\ X_Z^4[C_r^2 - X_Z^2]^{-9/2} - 90/120\ X_Z^2[C_r^2 - X_Z^2]^{-7/2}$,

$$N_z(6) = -945/720\, X_z^5[C_r^2 - X_z^2]^{-11/2} - 1050/720\, X_z^3[C_r^2 - X_z^2]^{-9/2} - 225/720\, X_z[C_r^2 - X_z^2]^{-7/2},$$

$$N_z(7) = -10395/5040\, X_z^6[C_r^2 - X_z^2]^{-13/2} - 14175/5040\, X_z^4[C_r^2 - X_z^2]^{-11/2} - 4725/5040\, X_z^2[C_r^2 - X_z^2]^{-9/2} - 225/5040\, [C_r^2 - X_z^2]^{-7/2},$$

$$N_z(8) = [-135135 X_z^7[C_r^2 - X_z^2]^{-15/2} - 21829 X_z^5[C_r^2 - X_z^2]^{-13/2} - 99225 X_z^3[C_r^2 - X_z^2]^{-11/2}]/40320 - 11025/40320\, X_z[C_r^2 - X_z^2]^{-9/2},$$

$$N_z(9) = [-2027025\, X_z^8[C_r^2 - X_z^2]^{-17/2} - 3783780\, X_z^6[C_r^2 - X_z^2]^{-15/2}]/362280 + [-2182950\, X_z^4[C_r^2 - X_z^2]^{-13/2} - 296900\, X_z^2[C_r^2 - X_z^2]^{-11/2} - 11025\, [C_r^2 - X_z^2]^{-9/2}]/362280,$$

$$N_z(10) = [-34459424 X_z^9[C_r^2 - X_z^2]^{-19/2} - 72972900 X_z^7[C_r^2 - X_z^2]^{-17/2}]/3622800 + [-51081030 X_z^5[C_r^2 - X_z^2]^{-15/2} - 13097700\, X_z^3[C_r^2 - X_z^2]^{-13/2} - 893025\, X_z[C_r^2 - X_z^2]^{-11/2}]/3622800,$$

The Taylor expansion for *ACOS* $[X_i/C_r]$ in powers of $(X_i - X_Z)$ is:

$$ACOS\,[X_i/C_r] = N_z(0) + N_z(1)(X_i - X_z) + N_z(2)(X_i - X_z)^2 + N_z(3)(X_i - X_z)^3 + N_z(4)(X_i - X_z)^4 + N_z(5)(X_i - X_z)^5 + N_z(6)(X_i - X_z)^6 + N_z(7)(X_i - X_z)^7 + N_z(8)(X_i - X_z)^8 + N_z(9)(X_i - X_z)^9 + N_z(10)(X_i - X_z)^{10} + N_z(11)(X_i - X_z)^{11}.$$

Applying the above coefficients to the power expansions of $(X_i - X_Z)$ in an interval phc, the equivalent expansions of *ACOS* $[X_i/C_r]$ in $(\zeta - \mu)$, in the interval is:

$$N_z(0) + N_z(1)(X_i - X_z) = N_z(1)[M_A(1,1) + M_A(1,2)(\zeta - \mu) + \ldots + M_A(1,39)(\zeta - \mu)^{38} + M_A(1,40)(\zeta - \mu)^{39}]$$

$$N_z(2)(X_i - X_z)^2 = N_z(2)[M_A(2,1) + M_A(2,2)(\zeta - \mu) + \ldots + M_A(2,39)(\zeta - \mu)^{38} + M_A(2,40)(\zeta - \mu)^{39}]$$

$$\vdots$$

$$N_z(10)(X_i - X_z)^{11} = N_z(10)[M_A(10,1) + M_A(10,2)(\zeta - \mu) + \ldots + M_A(10,39)(\zeta - \mu)^{38} + M_A(10,40)(\zeta - \mu)^{39}]$$

60_d) $${}_{phc}ACOS\,(X_i/C_r) = Q(phc,1) + Q(phc,2)(\zeta - \mu) + \ldots + Q(phc,39)(\zeta - \mu)^{38} + Q(phc,40)(\zeta - \mu)^{39}.$$

Where Q(phc,1) = Q(phc,1) + $N_z(0)$.

61 THE POWER EXPANSIONS FOR THE TORQUE EQUATION.

Each of the required terms and factors of the torque equation are now extant as sets of twentyfour 40 term polynomial expansions; i.e. there are a set of expansions for said terms and factors for each of the 24 equal intervals between 0 and π. These expansions are combined mathematically in the program to form twentyfour 40 term expansion equivalents to the torque equation in 54b3;

$$TRQ_{INST.} = dP\left([R + P_r SIN(G - A(59) + \sigma)][C_r^2 ACS(X_i/C_r) - X_i[C_r^2 - X_i^2]^{1/2}] + 2/3\, SIN\,\mu[C_r^2 - X_i^2]^{3/2}\right).$$

The expansions have the form:

61) $${}_{phc}TRQ = L(Phc,1) + L(Phc,2)(\zeta - \mu) + \ldots + L(phc,39)(\zeta - \mu)^{38} + L(phc,40)(\zeta - \mu)^{39}.$$

62 THE POWER EXPANSIONS FOR THE VOLUME EQUATION.

$$V_{INST.} = \int_{\theta_1}^{\theta_2}\left[R + P_r SIN(G - A(59) + \sigma)\right]\left(C_r^2 ACS(X_i/C_r) - X_i[C_r^2 - X_i^2]^{1/2}\right) + 2/3\, SIN\,\mu\,[C_r^2 - X_i^2]^{3/2}\, d\theta.$$

The volume integral above is modify to the integral 62 by changing from the RCS to the UCS; i.e.

62) $$V_{INST.} = 1/u\int_{\zeta_1 = u(\theta_1 - \beta)}^{\zeta_2 = u(\theta_2 - \beta)}[R + P_r SIN(G - A(59) + \sigma)](C_r^2 ACS(X_i/C_r) - X_i[C_r^2 - X_i^2]^{1/2}) + 2/3\, SIN\,\mu[C_r^2 - X_i^2]^{3/2}\, d\zeta.$$

The integrand in 62 can be now be replace by the expansion for torque in equation 61 above, and for each of the twenty-four intervals, Phc, and as μ is a constant d(ζ - μ) ≡ dζ the integral is:

62_a) $$VOL_{INST.} = 1/u\int_{\zeta_1 = \mu}^{\zeta_2 = u(\theta_2 - \beta)} L(Phc,1) + L(Phc,2)(\zeta - \mu) + \ldots + L(phc,39)(\zeta - \mu)^{38} + L(phc,40)(\zeta - \mu)^{39}\, d\zeta\ [\equiv d(\zeta - \mu)].$$

Integrating 62a interval by interval, the solution in an interval Phc is:

62_b) $$1/u\,[\,L(Phc,1)(\zeta - \mu) + (1/2)\,L(Phc,2)(\zeta - \mu)^2 + (1/3)\,L(Phc,2)(\zeta - \mu)^3 + \ldots + 1/40\,L(phc,40)(\zeta - \mu)^{40}\,]$$

Recognizing that the initial boundary of any interval phc is the end boundary of the preceding interval, that the integral solution without a constant is discontinuous at $\zeta_1 = \mu$, and the magnitude at any point in one interval must always be greater then any point in the preceding intervals, we add a constant at the beginning of each interval expansion equal to the preceding interval expansion's end magnitude; i.e. at μ. Keeping the form of our expansions consistent, we place the constant at the beginning of the integral solution expansion 62b above and term for term we designate the coefficients of the volume power expansion by: V(phc,n+ 1) = (1/u)L(phc,n)/(n) where n = 1 → 40, and

V(phc,1) = ${}_{(phc-1)}$VOL @ ζ = μ for phc = 2 → 24 and V(1,1) = 0.

The polynomial expansions for each interval in the annular cavity's volume varying regions have 41 terms and the form:

62$_c$) $_{phc}VOL = V(Phc,1) + V(Phc,2)(\zeta - \mu) + V(phc,3)(\zeta - \mu)^2 + \ldots + V(phc,41)(\zeta - \mu)^{40}$.

Once the expansions for the volume are acquired - and it takes less 40 seconds (including multiple disk transfers) from the beginning of the expansion generation program section to its end - all antecedent expansions become superfluous and unless tests for the expansions' errors are desired they are cleared from memory and storage. Use of the twenty-four, forty-one term expansions and the associated program for acquisition of the instant VVC volume at any point in the annular cavity doesn't noticeably hinder program speed.

63 EXPANSION EQUATION PRECISION.

Once the various power expansions for the terms, factors, torque and volume equations are generated, their accuracy can be easily tested. If an accuracy test is selected in the program for one of the topic expansions other then the volume expansion, its value is tested against the value acquired by direct evaluation of its equivalent equation at a very large number of incremented angles between 0 and π in the undulation coordinate system. Typical results of said tests are in table A below.

The volume expansions can also be tested. However, their percentage variation, are their deviation from the mean value of a volume increment, MIV, found utilizing the torque equation. The program uses the (UCS) CAAN coordinate system; i.e. $\Delta\theta = \zeta/u$.

63) INCREMENT'S MEAN VOLUME $= MIV_\zeta = (\Delta\zeta)[\,TRQ_{\zeta+\Delta\zeta/2} + TRQ_{\zeta-\Delta\zeta/2}\,]/2$

Two volume error tests are made. See table B below. One test is the maximum error of the volume from the beginning of the volume varying region of the undulation to CAAN angle ζ_n acquired by the volume power expansions, denoted here as $IVOl_\zeta$, compared to the summation of the mean increment values by 63 above in the same interval.

63$_a$) % ERROR OF UNDULATION REGION VOLUME TO $\zeta = 100\,[\Sigma\,MIV - IVOL_\zeta]/\Sigma\,MIV$

The second test is the percentage variation of the volume acquired for each increment by means of the power expansion compared to the mean value acquired by 63 above; i.e. the increment volume acquired by rotating the torque values at $\zeta_{n+\Delta\zeta/2}$ and $\zeta_{n-\Delta\zeta/2}$ through angle $\Delta\zeta$ and taking their mean value.

63$_b$) % ERROR OF INCREMENTS VOLUME AT ANY $\zeta = 100\,[MIV_\zeta - (IVOL_{\zeta+\Delta\zeta/2} - IVOL_{\zeta-\Delta\zeta/2})]/MIV_\zeta$.

EXPANSION EQUATIONS ERROR REPORT

24 spaced sequential intervals where each function is a 40 term expansion.

PERCENT ERROR = 100 [FUNCTION VALUE - EXPANSION VALUE]/FUNCTION VALUE

Engine: CIR4 cycle, Displacement ≈ 125 CU.I., COMPRESSION RATIO: 15.7

PROGRAM : COMPUTER PROGRAMMING FOR DESIGN OF FRASCA ROTARY ENGINES ©

Language: POWERBASICS™V3.1
PowerBASIC®, Carmel, CA 93923

Computer: 486DX4 - 100 and 12 meg.

TABLE A Equation ELEMENT	NUMBER OF INCREMENTS	MAXIMUM PERCENT ERROR	MEAN PERCENT ERROR
$\alpha/2\,[1 - COS\,u(\theta - ß)]$	31439	$\approx 7.5 \times 10^{-10}$	$< 5.4 \times 10^{-14}$
$X_i = ec + P_r SIN(P_a - \sigma)$	31439	$\approx 5.0 \times 10^{-14}$	$< 2.9 \times 10^{-14}$
X_i^2	31439	$\approx 7.0 \times 10^{-14}$	$< 4.0 \times 10^{-14}$
$[\,C_r + X_i\,]^{-1/2}$	31439	$\approx 8.4 \times 10^{-11}$	$< 4.5 \times 10^{-12}$
$[\,C_r - X_i\,]^{-1/2}$	31439	$\approx 6.6 \times 10^{-11}$	$< 1.2 \times 10^{-12}$
$[\,C_r^2 - X_i^2\,]^{-1/2}$	31439	$\approx 8.7 \times 10^{-11}$	$< 5.7 \times 10^{-12}$
$ACS\,(X_i/C_r)$	31439	$\approx 5.6 \times 10^{-11}$	$< 9.2 \times 10^{-13}$
$R + P_r SIN(G - P_a + \sigma)$	31439	$\approx 4.3 \times 10^{-13}$	$< 9.1 \times 10^{-14}$
TORQUE Equation	31439	$\approx 4.9 \times 10^{-9}$	$< 3.0 \times 10^{-10}$
TABLE B VOLUME EXPANSION	INCREMENT $0 \le$ CAAN $\le \pi$	MAXIMUM % VOLUME ERROR 0 TO θ	MAXIMUM % INCREMENT ERROR
IVOL	30000	$< 2.4 \times 10^{-7}$	$< 2.4 \times 10^{-7}$

64 THERMODYNAMIC REVIEW

Here is a very short review of the salient thermodynamic relationships used to determine the power characteristics of the FRE engines. Text books by G. J. Van Wylen[38] and Mark W. Zemansky[42] along with others* are relied upon for reference. * Others includes references: 1, 3a, 8a, 17, 19, 20, 22, 22a, 23, 31a, 31b, 33, 37, 38, 40. Symbols and comments:

- ϱ is density in lbm/ft^3,
- P is pressure in lbf/ft^2,
- Q indicates thermal energy in BTU = 778.26 ft - lbf
- lbm indicates pound mass,
- R° = F° + 459.69°,
- lbf indicates a pound force.
- v is specific volume in ft^3/lbm,
- W indicates work in lbf - ft.
- g_c is the gravitation conversion constant: $(32.17\ lbm\text{-}ft/sec^2)/lbf = 1$
- g acceleration of gravity; i.e. 32.17 ft/sec^2
- δ indicates an inexact differential. (value dependent on path.)
- Lower case letters generally mean a lbm bases.

The closed system energy equation, where U is internal energy, z is feet above a reference and V is velocity (ft/sec) is:

64) $$\delta Q = dU + d(mV^2/g_c)/2 + d(z\ mg/g_c) + \delta W$$

The open system energy equation, where m_i, m_e and m_s, are mass in, out (lbm/sec) and system mass respectively, is:

64_a) $$\delta m_i[u_i + P_iv_i + V_i^2/(2g_c) + z_i] + \delta Q_i = d[U_s + m_sV_s^2/(2g_c) + m_sZ_s] + \delta W_o + \delta m_e[u_e + P_ev_e + V_e^2/(2g_c) + z_e].$$

Enthalpy on the mole and lbm bases are respectively:

64_b) $$H = U + PV_{sv},$$ 64_{b1}) $$h = u + Pv,$$

and in the differential form:

64_{b2}) $$dH = dU + PdV_{sv} + V_{sv}\ dP,$$ 64_{b3}) $$dh = du + P\ dv + v\ dP,$$

using 64b1 in equation 64a, we have:

64_c) $$\delta m_i[h_i + V_i^2/(2g_c) + gz_i/g_c] + \delta Q_i = d[U + mV^2/(2g_c) + mgZ/g_c] + \delta W_o + \delta m_e[h_e + V_e^2/(2g_c) + gz_e/g_c].$$

Entropy for a reversible process is:

64_d) $$Tds = \delta Q = dU + d(mV^2/g_c)/2 + d(z\ mg/g_c) + \delta W,$$

and for processes with irreversibilities this becomes:

64_{d1}) $$Tds = \delta Q = dU + d(mV^2/g_c)/2 + d(z\ mg/g_c) + \delta W + \text{Lost Work}.$$

When ignoring elevation and kinetic energy changes during a process, (i.e. $d(mgz/g_c) = 0$ and $d(mV^2/g_c)/2 = 0$), formulae 64d and 64d1 for a close system become:

64_{d2}) $$T\ dS = \delta Q = dU + \delta W,$$ and 64_{d3}) $$T\ dS = \delta Q = dU + \delta W, + LW.$$

In a close system, work is done at the boundary (i.e. δW= Pdv) and 64d2 on a molar and lbm basis respectively is:

64_e) $$T\ dS = \delta Q = dU + P\ dV,$$ and 64_{e1}) $$T\ ds = \delta q = du + P\ dv.$$

Combining 64e and 64e1 with 64b2 and 64b3 we have:

64_f) $$T\ dS = H - V\ dP$$ 64_{f1}) $$T\ ds = dh - v\ dP$$

In steady flow conditions (i.e. $\delta m_i = \delta m_e$) the closed system term in 64c is:

$$d[U + mV^2/(2g) + mgZ/g_c] = 0,$$

and 64c on a unite mass and differential form with rearrangement is:

$$(h_e - h_i) + [V_e^2/(2g_c) - V_i^2/(2g_c)] + (z_e - z_i)g/g_c + \delta W_o - [\delta Q_i = TdS] = 0$$

64_g) $$dh + d[V_i^2/(2g_c)] + g/g_c\ dz_i + \delta W_o - T\ ds = 0,$$

$$[T\ ds + v\ dP] + d[V_i^2/(2g_c)] + g/g_c\ dz_i + \delta W_o - T\ ds = 0,$$

64_h) $$v\ dP + d[V_i^2/(2g)] + g/g_c\ dz_i + \delta W = 0$$ {Bernoulli's equation].

With irreversibilities this becomes:

64_i) $$v\ dP + d[V_i^2/g_c] + g/g_c\ dz_i + \delta W + \text{lost work} = 0.$$

65 PERFECT GAS RELATIONSHIPS.

65) $$Pv = RT.$$

Avogadro's law: EQUAL VOLUMES OF DIFFERENT IDEAL GASES AT THE SAME TEMPERATURE AND PRESSURE CONTAIN THE SAME NUMBER OF MOLECULES.

65_a) $$P\ V = n\ M\ R\ T,$$

where "n" is the number of moles, V is the volume, and M is the molecular weight. Applying Avogadro to two different ideal gases with mole masses, M_a and M_b, we have: