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Using  $X = C_r \cos \Omega$  and  $dX = -C_r \sin \Omega d\Omega$ , Copyright © 1998 by Joseph F. Frasca

$$53_{b1}) \quad \text{PART Area} = \int_0^* -2C_r^2 \sin^2 \Omega d\Omega = \int_0^* C_r^2 [-\Omega + 1/2 \sin 2\Omega] = C_r^2 \text{ACS}(X/C_r) - X_i [C_r^2 - X_i^2]^{1/2}.$$

The partition area equation is then:

$$53_{b2}) \quad \text{PART Area} = C_r^2 \text{ACS}(X/C_r) - X_i [C_r^2 - X_i^2]^{1/2} \equiv C^2 \text{ATN}([C_r^2 - X_i^2]^{1/2}/X) - X_i [C_r^2 - X_i^2]^{1/2}.$$

At maximum partition extension into the cavity  $X_i = X_{\min}$ ; therefore, the maximum partition cavity area is:

$$53_{b3}) \quad \text{PART Area}_{\max} = C_r^2 \text{ACS}(X_{\min}/C_r) - X_{\min} [C_r^2 - X_{\min}^2]^{1/2}.$$

Using the program entry for the desired VVC volume ratio, AR, and the value of  $\text{PART Area}_{\max}$ , the program increases the value of  $X_i$  in 53b1 by iteration from its initial minimum value  $X_{\min}$  until the ratio  $\text{PART Area}_{\max} / \text{PART Area} \geq \text{AR}$ ; i.e. the program incrementally decreases the partition angular displacement into the cavity until the desired AR is achieved. The constant  $X_m$  is defined as the value of  $X_i$  when the desired ratio is attained; i.e.  $X_m = X_i$  when  $\text{PART Area}_{\max} / \text{PART Area} \geq \text{AR}$ . We now find the partition's angular displacement  $\alpha$  to achieve this minimum area.

Although the relative orientations of the various elements,  $S_r$ ,  $P_r$ , circle center etc, remain at all times the same, in the present design,  $S_r$  becomes shorter and the PCS origin rotates slightly clockwise while the bisecting points of both the cavity arc and cord move slightly.

At minimum partition cavity area, using  $U_m = (X_m - e_c)$  and  $V_m = (C_r - X_m + e_c)$ , and with reference to 53a and 53b,

$$53_{c}) \quad S_r = [P_r^2 - (X_m - e_c)^2]^{1/2} = [P_r^2 - U_m^2]^{1/2},$$

$$53_{c1}) \quad \Gamma_m = \text{ATN}[(C_r - X_m + e_c)/S_r],$$

With  $\delta_m = \text{ATN}(e_c/S_r)$ , the actual partition angular displacement,  $\alpha_2$ , between minimum and maximum areas is:

$$53_{c2}) \quad \alpha_2 = e_c - (\Gamma_m - \delta_m).$$

The minimum extension into the cavity is :

$$53_{c3}) \quad \alpha_1 = (\Gamma_m - \delta_m), \text{ where } \alpha_1 + \alpha_2 = e_c.$$

Using the equation of partition angular extension,  $\phi$ , into the annular cavity,

$$53_{c4}) \quad \phi = \alpha_1 + (\alpha_2/2)[1 - \cos u(\theta - \beta)],$$

the major parameter  $X_i$ , the distance between the PCS origin and the face surface is:

$$53_{d}) \quad X_i = P_r \sin(P_a - \phi) + e_c.$$

For clutter control  $A(59) = P_a - \alpha_1$ , and  $\sigma = (\alpha_2/2)[1 - \cos u(\theta - \beta)]$ .

$$53_{d1}) \quad X_i = P_r \sin(A(59) - (\alpha_2/2)[1 - \cos u(\theta - \beta)]) + e_c = P_r \sin(A(59) - \sigma) + e_c.$$

With the equation for  $X_i$  dependent on  $\theta$ , we derive  $\theta$  dependent equations for partition area, volume and arc length h.

$$53_{d3}) \quad \text{AREA}_{\text{INST.}} = C_r^2 \text{ACS}(X/C_r) - X_i [C_r^2 - X_i^2]^{1/2}.$$

Discussion will be limited to engines that have partitions with constant thickness,  $t$ . The annular cavity volume of said partitions at  $\theta$  and the gap length, are given by:

$$53_{d4}) \quad \text{PARVOL}_{\text{INST.}} = t [C_r^2 \text{ACS}(X/C_r) - X_i [C_r^2 - X_i^2]^{1/2}], \text{ and}$$

$$53_{d5}) \quad \text{GAP LENGTH}_{\text{INST.}} = 2 C_r \text{ACS}(X/C_r).$$

Although not covered in this discussion, this engine with tapered partitions would have its partition's thickness inversely proportional to the distance from the pivot point, as in the first engine designs discussed.

### PARTITION ACCELERATION

The partition acceleration follows from 53d3 with:

$$\sigma = (\alpha_2/2)[1 - \cos u(\theta - \beta)],$$

$$d\sigma/dt = d\sigma/d\theta d\theta/dt = u (\alpha_2/2) [\sin u(\theta - \beta)] d\theta/dt, \text{ and}$$

$$d^2\sigma/dt^2 = u^2 \alpha_2/2 \cos u(\theta - \beta) (d\theta/dt)^2 + u \alpha_2/2 \sin u(\theta - \beta) d^2\theta/dt^2,$$

which we approximated with:

$$d^2\sigma/dt^2 \approx (1.1) u^2 (\alpha_2/2) [\cos u(\theta - \beta)] (d\theta/dt)^2.$$

### 54 CIR ENGINE INSTANT TORQUE EQUATION.

With reference to figures 54, 54a, and 54b the pressure differential,  $dP$ , on a partition's area element,  $dA$ , located at point  $(x_m, y_n)$  in the PCS acts at moment arm distance,  $M_A$ , from the rotor axis to impart a moment to the rotor. The moment arm at  $(\theta, x_m, y_n)$  is:

$$54) \quad M_A = R + x_m \sin \mu + y_n \sin \nu + P_r \sin(G - A(59) + \sigma).$$

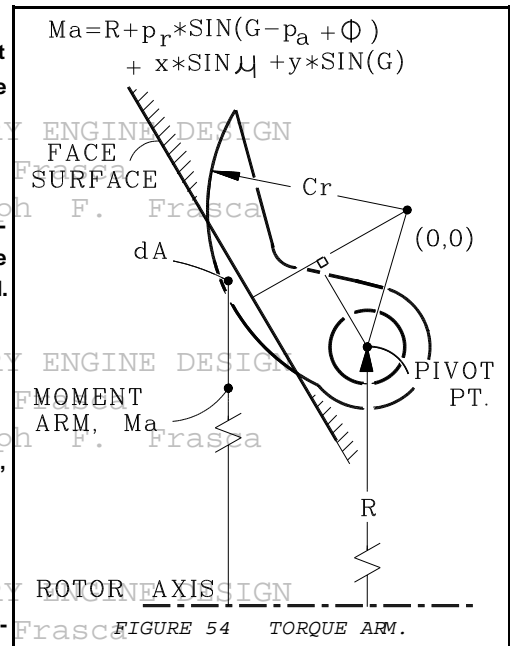


FIGURE 54 TORQUE ARM.