430$_a$)

$$M_{(PART\ X\text{-}Z)\text{-}X} = \int_0^{\phi_i} P_{xzx7}\sigma^3 SIN(G + \phi_i - \sigma) + P_{xzx6}\sigma^2 COS(G + \phi_i - \sigma) + P_{xzx5}\sigma^2 SIN(G + \phi_i - \sigma) + [P_{xzx4}\sigma COS(G + \phi_i - \sigma) + P_{xzx3}\sigma SIN(G + \phi_i - \sigma) + P_{xzx2} COS(G + \phi_i - \sigma) + P_{xzs1} SIN(G + \phi_i - \sigma)] COS\theta_i .$$

30$_b$)

$$M_{(PART\ X\text{-}Z)\text{-}X} = [P_{xzx7}\phi_i^3 SING + P_{xzx6}\phi_i^2 COSG + P_{xzx5}\phi_i^2 SING + P_{xzx4}\phi_i COSG] COS\theta_i + [P_{xzx3}\phi_i SING + P_{xzx2} COSG + P_{xzx1} SING - P_{xzx2} COS(G + \phi_i) - P_{xzx1} SIN(G + \phi_i)] COS\theta_i .$$

## 31 THE WAVE SURFACE X-Z PLANE PROJECTION AREA'S X-AXIS MOMENT.

The moment about the X-axis of the wave surface X-Z projection area is the combination of the face surface X-Z projection area's X-axis moment, 30j, and the boundary partitions X-Z projection areas moments about the X-axis; i.e.

31$_a$)

$$M_{(WAVE\ XZ)\text{-}X} = M_{FACE\ X\text{-}Z)\text{-}X} \pm \left[M_{(PART\ X\text{-}Z)\text{-}X\ @\ \theta_i + \text{ß}} + M_{(PART\ X\text{-}Z)\text{-}X\ @\ \theta_i}\right].$$

## 32 THE FACE SURFACE PROJECTION IN THE Y-Z PLANE.

The equation for a VVC's face projection in the Y-Z plane, equation 32 below, is very similar to that for its X-Z plane projection. The dZ and d$\phi$ integrands in both the topic equations 27 and 32 are identical. The variation between the equations is in their projection factor. The projection factors in 27 and 32 are respectively SIN$\theta$ and COS$\theta$.

32)

$$A_{Y\text{-}Z} = \int_{\theta_1}^{\theta_2} \int_{aCOSG}^{(a+N\phi)COSG} [R + Z\ TANG] COS\theta\ dZ\ d\theta + \int_{\theta_1}^{\theta_2} \int_0^{\phi_{\theta i}} [R + aSIN(G + \phi)] aSIN(G + \phi)\ d\phi\ COS\theta\ d\theta$$

Using the constants generated in section 27 above 32 becomes:

32$_a$)

$$_{facesurf}A_{Y\text{-}Z} = \int S_a COS\theta + S_b COSu(\theta - \text{ß}) COS\theta + S_c COS^2 u(\theta - \text{ß}) COS\theta + S_d COS^3 u(\theta - \text{ß}) COS\theta\ d\theta.$$

Ho hum, back to the math appendix 67 - 70 for $\int COS^n u(\theta - \text{ß}) COS\theta d\theta$ solutions of the terms in 32a and collecting coefficients:

$$[S_a + S_c/2] SIN\theta_i = A_{YZ1} SIN\theta_i, \quad [-r_2 S_b + 6 S_d u^2 r_3 r_2] COSu(\theta_i - \text{ß}) SIN\theta_i = A_{YZ2} COSu(\theta_i - \text{ß}) SIN\theta_i,$$

$$[S_b r_2 u - 6u^3 S_d r_3 r_2] SINu(\theta_i - \text{ß}) COS\theta_i = A_{YZ3} SINu(\theta_i - \text{ß}) COS\theta_i, \quad [-s_2 S_c/2] COS2u(\theta_i - \text{ß}) SIN\theta_i = A_{YZ4} COS2u(\theta_i - \text{ß}) SIN\theta_i,$$

$$[us_2 S_c] SIN2u(\theta_i - \text{ß}) COS\theta_i = A_{YZ5} SIN2u(\theta_i - \text{ß}) COS\theta_i, \quad [S_d r_3] COS^3 u(\theta_i - \text{ß}) SIN\theta_i = A_{YZ6} COS^3 u(\theta_i - \text{ß}) SIN\theta_i,$$

and $[-3uS_d r_3] COS^2 u(\theta_i - \text{ß}) SINu(\theta_i - \text{ß}) SIN\theta_i = A_{YZ7} COS^2 u(\theta_i - \text{ß}) SINu(\theta_i - \text{ß}) SIN\theta_i$ .

32$_c$)

$$\int_{\theta_i}^{\theta_i + A(11)} AREA_{(FACE\ SURFACE\ Y\text{-}Z)} = A_{YZ1} SIN\theta_i + A_{YZ2} COSu(\theta_i - \text{ß}) SIN\theta_i + A_{YZ3} SINu(\theta_i - \text{ß}) COS\theta_i + A_{YZ4} COS2u(\theta_i - \text{ß}) SIN\theta_i + A_{YZ5} SIN2u(\theta_i - \text{ß}) COS\theta_i + A_{YZ6} COS^3 u(\theta_i - \text{ß}) SIN\theta_i + A_{YZ7} COS^2 u(\theta_i - \text{ß}) SINu(\theta_i - \text{ß}) SIN\theta_i.$$

## MINIMUM AND MAXIMUM FACE SURFACE PROJECTIONS ON Y-Z PLANE

For clutter control: $A(58) = a + N\alpha_1$, $A(59) = a + N(\alpha_1 + \alpha_2)$, $R_{z1} = A(58)COSG$, $R_{z2} = A(59)COSG$, $R_{z0} = aCOSG$

Using equation 32 above:

32$_d$)

$$AREA_{YZ\ min} = \int_{R_{z0}}^{R_{z1}} \int RZ + Z^2/2\ TAN\ G + \int_0^{\phi_1} - Ra\ COS(G + \phi) + a^2/2\ [\phi - 1/2\ SIN\ 2(G + \phi)] COS\ \theta\ d\theta .$$

32$_e$)

$$AREA_{YZ\ min} = -\int_{\theta_1}^{\theta_2} \left[R(R_{z1} - R_{z0}) + TANG[R_{z1}^2 - R_{z0}^2]/2 - Ra[COS(G + \alpha_1) - COS\ G] + a^2/2\ [\alpha_1 - SIN2(G + \alpha_1) + SIN2G\right] SIN\theta.$$

32$_f$)

$$AREA_{YZmax} = -\int_{\theta_1}^{\theta_2} [R(R_{z2} - R_{z0}) + TANG[R_{z2}^2 - R_{z0}^2]/2 - Ra[COS(G + \alpha_1 + \alpha_2) - COSG] + a^2/2\ [\alpha_1 + \alpha_2 - SIN2(G + \alpha_1 + \alpha_2) + SIN2G\big] SIN\theta.$$

## 33 THE PARTITION Y-Z PLANE PROJECTION.

The projection areas on the Y-Z plane of the VVC's boundary partitions is similar to that in section 28; however, the *COS*$\theta$ factor is replaced with SIN $\theta$ as indicated in 33 below.

33)

$$AREA_{(PART\ Y\text{-}Z)} = \left[aN/2\ \phi_{\theta i}^2 + N^2/6\ \phi_{\theta i}^3\right] SIN\theta.$$

A VVC's wave surface projection in the Y-Z plane is, as in the X-Z plane, a combination of the boundary partition's projection area and face surface projection area.

## 34 THE FACE SURFACE Y-Z PLANE PROJECTION'S Y-AXIS MOMENT.

For the moment about the Y-axis of the VVC Y-Z plane projection, the methods used in section 29 for the moment about the X-axis of the VVC X-Z plane projection are followed. The integrations are identical but for the last in d$\theta$.

$$34)\quad M_{(FACE\ Y\text{-}Z)\text{-}Y} = \int_{\theta_1}^{\theta_2}\int_{aCOSG}^{(a+N\phi)COSG} [R+ Z\,TANG][Z]COS\theta\, dZ\, d\theta + \int_{\theta_1}^{\theta_2}\int_{0}^{\phi_{\theta i}} [R + aSIN(G+\phi)][aCOS(G+\phi)]aSIN(G+\phi)\, d\phi\, COS\theta\, d\theta.$$

$\downarrow$*{MOMENT ARM]* $\quad\downarrow$*{MOMENT ARM]*

Using the constants generated in 29 for the $d\theta$ integral, the final integral, and changing the projection factor from $SIN\theta$ to $COS\theta$ in 29i, the integrals in equation 34 becomes:

$$34_b)\quad \int M_oCOS\theta + M_1COSu(\theta - ß)COS\theta + M_2COS^2u(\theta - ß)COS\theta + M_3COS^3u(\theta - ß)COS\theta + M_4COS^4u(\theta - ß)COS\theta d\theta.$$

Using the math appendix solutions in 35b and collecting common coefficients:

$[M_o + M_2/2 - 6\ M_4u^2r_4]SIN\theta = M_{YZY0}SIN\theta$, $\quad [M_1r_2u - 6\ M_3u^3r_3r_2]SINu(\theta - ß)COS\theta = M_{YZY1}SINu(\theta - ß)COS\theta$,
$[- M_1r_2 + 6M_3u^3r_3r_2]COSu(\theta\text{-}ß)SIN\theta = M_{YZY2}COSu(\theta\text{-}ß)SIN\theta$, $[M_2S_2u - 12\ u^3r_4s_2M_4]SIN2u(\theta\text{-}ß)COS\theta = M_{YZY3}SIN2u(\theta - ß)COS\theta$,
$[- M_2s_2/2 + 6M_4u^2r_2s_2]COS2u(\theta\text{-}ß)SIN\theta = M_{YZY4}COS2u(\theta\text{-}ß)SIN\theta$, $\quad [M_3r_3]COS^3u(\theta - ß)SIN\theta = M_{YZY5}COS^3u(\theta - ß)SIN\theta$,
$[M_4r_4]COS^4u(\theta\text{-}ß)SIN\theta = M_{YZY6}COS^4u(\theta\text{-}ß)SIN\theta$, $[- 3uM_3r_3u]COS^2u(\theta\text{-}ß)SINu(\theta\text{-}ß)COS\theta = M_{YZY7}COS^2u(\theta\text{-}ß)SINu(\theta\text{-}ß)COS\theta$,
and $\quad [- 4uM_4r_4]COS^3u(\theta - ß)SINu(\theta - ß)COS\theta = M_{YZY8}COS^3u(\theta - ß)SINu(\theta - ß)COS\theta$.

The Y-axis moment of a VVC's face surface projection on the Y-Z plane is:

$$34_c)\quad \ast_{\theta_i}^{\theta_i + A(11)} M_{(FACEY\ Y\text{-}Z)\text{-}Y} = M_{YZY0}SIN\theta + M_{YZY1}SINu(\theta - ß)COS\theta + M_{YZY2}COSu(\theta - ß)SIN\theta + M_{YZY3}SIN2u(\theta - ß)COS\theta$$
$$+ M_{YZY4}COS2u(\theta - ß)SIN\theta + M_{YZY5}COS^3u(\theta - ß)SIN\theta + M_{YZY6}COS^4u(\theta - ß)SIN\theta$$
$$+ M_{YZY7}COS^2u(\theta - ß)SINu(\theta - ß)COS\theta + M_{YZY8}COS^3u(\theta - ß)SINu(\theta - ß)COS\theta.$$

## MINIMUM AND MAXIMUM Y-Z AREA MOMENT ABOUT THE Y-AXIS

For clutter control: $A(58) = a + N\alpha_1$, $A(59) = a + N(\alpha_1 + \alpha_2)$, $R_{z1} = A(58)COSG$, $R_{z2} = A(59)COSG$, $R_{z0} = aCOS\ G$

Using 34 above:

$$34_d)\quad M_{(FACE\ Y\text{-}Z)\text{-}Y} = \int_{\theta_1}^{\theta_2}\int_{R_{z0}}^{R_{z1}} RZ^2/2 + Z^3/3\ TAN\ G + \int_{\theta_1}^{\theta_2}\ast_{0}^{\alpha_1} Ra^2/2\ SIN^2(G+\phi) + a^3/3\ SIN^3(G+\phi)COS\theta\, d\theta.$$

$$34_e)\quad {}_{MIN}M_{(FACE\ Y\text{-}Z)\text{-}Y} = \ast_{\theta_1}^{\theta_2} - \Big[R\,[R_{z1}{}^2 - R_{z0}{}^2]/2 + [R_{z1}{}^3 - R_{z0}{}^3]/3\ TANG + Ra^2/2\ [SIN^2(G+\alpha_1) - SIN^2G] + a^3/3\ SIN^3(G+\alpha_1)\Big]SIN\theta.$$

$$34_f)\quad {}_{MAX}M_{(FACE\ Y\text{-}Z)\text{-}Y} = \ast_{\theta_1}^{\theta_2} - \Big[R\,[R_{z2}{}^2 - R_{z0}{}^2]/2 + [R_{z2}{}^3 - R_{z0}{}^3]/3\ TANG + Ra^2/2\ [SIN^2(G+\alpha_1+\alpha_2) - SIN^2G] + a^3/3\ SIN^3(G+\alpha_1+\alpha_2)\Big]SIN\theta.$$

## 35 THE PARTITION Y-Z PLANE PROJECTION'S Y-AXIS MOMENT.

With exception of the projection factor, the partition's y-z projection area y-axis moment is identical to the partitions x-z projection area x-axis moment. The constants are then the same; i.e. $P_{yzy0} = P_{xzx0}$, $P_{yzy1} = P_{xzx1}$, . . . , $P_{yzy7} = P_{xzx7}$.

$$35)\quad M_{(PART\ Y\text{-}Z)\text{-}Y} = \int_{0}^{\phi_i}\int_{a}^{a+N\phi} [rCOS(G + \phi_i - \sigma)][r\, d\sigma\, dr][SIN\theta_i] = \int_{0}^{\phi_i} [a^2N\sigma + aN^2\sigma^2 + N^3\sigma^3/3]COS(G + \phi_i - \sigma)SIN\ \theta_i\, d\sigma.$$

$\downarrow$*{MOMENT ARM][PROJ. ADJ.}*$\downarrow$ $\quad\uparrow$*{dA]*

$$35_a)\quad M_{(PART\ Y\text{-}Z)\text{-}Y} = [P_{YZY7}\phi_i^3SING + P_{YZY6}\phi_i^2COSG + P_{YZY5}\phi_i^2SING + P_{YZY4}\phi_iCOSG]SIN\theta_i$$
$$+ [P_{YZY3}\phi_iSING + P_{YZY2}COSG + P_{YZY1}SING - P_{YZY2}COS(G+\phi_i) - P_{YZY1}SIN(G+\phi_i)]SIN\theta_i.$$

## 36 Y-Z AREA PROJECTION'S Y-AXIS MOMENT.

Equations 34c and 35a are combined for the VVC wave surface Y-Z projection's moment about the Y-axis.

$$36)\quad {}_YM_{WAVEYZ} = M_{FACE\ YZ\text{-}Y} \pm \big[[M_{(PART\ Y\text{-}Z)\text{-}Y}]_{\theta_i+ß} + [M_{(PART\ Y\text{-}Z)\text{-}Y}]_{\theta_i}\big].$$

The engine design computer program, using the VVCs' pressures, determines the X-axis, Y-axis and Z-axis collection of forces and X-axis and Y-axis moments at the rotor co-ordinate system origin, also the maximum and mean radial and thrust loads and the maximum and mean X-axis and Y-axis moments.

## 37 CAVITY'S Y-AXIS MOMENT.

The center of mass of the annular cavity void is now found so that the rotor will be balanced when formed. With a ceramic rotor wave surface the greater the accuracy of these equation the less post form machining and balancing required.

With $B = G + \phi_{\theta i} = A(7) + \alpha_1 + \alpha_2/2\ [\ 1 - COS\epsilon(\theta_i - ß)\ ]$ and with reference to figure 37 the Y centroid is found with:

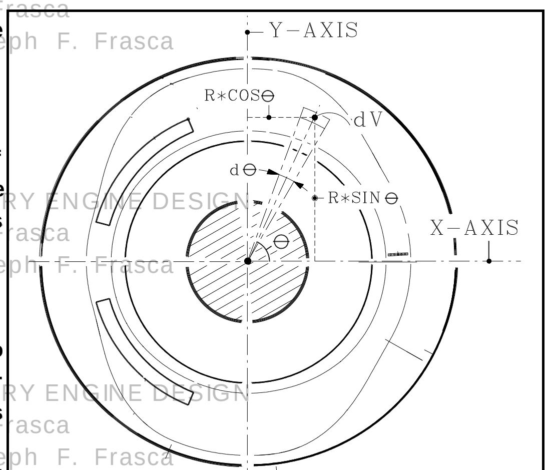


*FIGURE 37 ANNULAR CAVITY CENTROIDS.*

**37)** ↓{*MOMENT ARM*] ↓{*VOLUME ELEMENT, dV*]

$$\mathbf{CENT_Y} = \int_{\theta_1}^{\theta_2}\int_0^{\phi_i}\int_a^{r=a+N\sigma}\Big[[R + r\,SIN(B-\sigma)]\,COS\theta\Big]\Big[[R + r\,SIN(B-\sigma)]\,d\theta\, r d\sigma\, dr\Big] = \int_{\theta_1}^{\theta_2}\int_0^{\phi_i}\int_a^{r=a+N\sigma}[rR^2 + 2Rr^2 SIN(B-\sigma) + r^3 SIN^2(B-\sigma)]\,d\sigma\, dr\; COS\theta\, d\theta.$$

**37$_a$)**
$$\mathbf{CENT_Y} = \int_{\theta_1}^{\theta_2}\int_0^{\phi_i} R^2/2\;[2aN\sigma + N^2\sigma^2] + 2R/3\; SIN(B-\sigma)[3a^2N\sigma + 3aN^2\sigma^2 + N^3\sigma^3]\,d\sigma\; COS\theta\, d\theta$$
$$+ \int_{\theta_1}^{\theta_2}\int_0^{\phi_i}[1/8 - 1/8\; COS2(B-\sigma)][4a^3N\sigma + 6a^2N^2\sigma^2 + 4aN^3\sigma^3 + N^4\sigma^4]\,d\sigma\; COS\theta\, d\theta.$$

**For clutter control:**

$[aR^2N + a^3N/2]\sigma = A_1\sigma$, $[R^2N^2/2 + 3/4\; a^2N^2]\sigma^2 = A_2\sigma^2$, $[aN^3/2]\sigma^3 = A_3\sigma^3$, $[N^4/8]\sigma^4 = A_4\sigma^4$,
$[2Ra^2N]\sigma SIN(B-\sigma) = A_5\sigma SIN(B-\sigma)$, $[2RaN^2]\sigma^2 SIN(B-\sigma) = A_6\sigma^2 SIN(B-\sigma)$, $[2/3\; RN^3]\sigma^3 SIN(B-\sigma) = A_7\sigma^3 SIN(B-\sigma)$,
$[-1/2\; a^3N]\sigma COS2(B-\sigma) = A_8\sigma COS2(B-\sigma)$, $[-3/4\; a^2N^2]\sigma^2 COS2(B-\sigma) = A_9\sigma^2 COS2(B-\sigma)$,
$[-1/2\, aN^3]\sigma^3 COS2(B-\sigma) = A_{10}\sigma^3 COS2(B-\sigma)$, $[-1/8\; N^4]\sigma^4 COS2(B-\sigma) = A_{11}\sigma^4 COS2(B-\sigma)$.

**37$_b$)**
$$_{VCAV}\mathbf{CENT_Y} = \int_{\theta_1}^{\theta_2}\int_0^{\phi_i}\Big[A_1\sigma + A_2\sigma^2 + A_3\sigma^3 + A_4\sigma^4 + A_5\sigma SIN(B-\sigma) + A_6\sigma^2 SIN(B-\sigma) + A_7\sigma^3 SIN(B-\sigma)\Big]d\sigma\; COS\theta\, d\theta$$
$$+ \int_{\theta_1}^{\theta_2}\int_0^{\phi_i}\Big[A_8\sigma COS2(B-\sigma) + A_9\sigma^2 COS2(B-\sigma) + A_{10}\sigma^3 COS2(B-\sigma) + A_{11}\sigma^4 COS2(B-\sigma)\Big]d\sigma\; COS\theta\, d\theta.$$

**Referring to the math appendix 40 - 43 for solutions of $\int\sigma^n SIN(B-\sigma)d\sigma$ and 48 -51 for $\int\sigma^n COS2(B-\sigma)d\sigma$ the above integrands' terms integrate to:**

$\int[A_1\sigma + A_2\sigma^2 + A_3\sigma^3 + A_4\sigma^4]\,d\sigma = 1/2\; A_1\sigma^2 + 1/3\; A_2\sigma^3 + 1/4\; A_3\sigma^4 + 1/5\; A_4\sigma^5$,
$A_5\int\sigma\, SIN(B-\sigma)\,d\sigma = A_5\sigma COS(B-\sigma) + A_5 SIN(B-\sigma)$,
$A_6\int\sigma^2 SIN(B-\sigma)\,d\sigma = A_6\sigma^2 COS(B-\sigma) + 2A_6\sigma SIN(B-\sigma) - 2A_6 COS(B-\sigma)$,
$A_7\int\sigma^3 SIN(B-\sigma)\,d\sigma = A_7\sigma^3 COS(B-\sigma) + 3A_7\sigma^2 SIN(B-\sigma) - 6A_7\sigma COS(B-\sigma) - 6A_7 SIN(B-\sigma)$,
$A_8\int\sigma COS2(B-\sigma)\,d\sigma = A_8[-\sigma/2\; SIN2(B-\sigma) + 1/4\; COS2(B-\sigma)]$,
$A_9\int\sigma^2 COS2(B-\sigma)d\sigma = A_9[-\sigma^2/2\; SIN2(B-\sigma) + \sigma/2\; COS2(B-\sigma) + 1/4\; SIN2(B-\sigma)]$,
$A_{10}\int\sigma^3 COS2(B-\sigma)d\sigma = A_{10}[-\sigma^3/2\; SIN2(B-\sigma) + 3/4\;\sigma^2 COS2(B-\sigma) + 3/4\;\sigma SIN2(B-\sigma) - 3/8\; COS\,2(B-\sigma)]$,
$A_{11}\int\sigma^4 COS2(B-\sigma)d\sigma = A_{11}[-\sigma^4/2\; SIN2(B-\sigma) + \sigma^3 COS2(B-\sigma) + 3\sigma^2/2\; SIN2(B-\sigma) - 3\sigma/2\; COS2(B-\sigma) - 3/4\; SIN2(B-\sigma)]$.

**Collecting coefficients of like orders of σ, we have:**

$[1/2\; A_1]\sigma^2 = B_1\sigma^2$, $[1/3A_2]\sigma^3 = B_2\sigma^3$, $[1/4\; A_3]\sigma^4 = B_3\sigma^4$, $[1/5\; A_4]\sigma^5 = B_4\sigma^5$,
$[A_5 - 6A_7]SIN(B-\sigma) = B_5 SIN(B-\sigma)$, $[-2A_6]COS(B-\sigma) = B_6 COS(B-\sigma)$, $[2A_6]\sigma SIN(B-\sigma) = B_7\sigma SIN(B-\sigma)$,
$[A_5 - 6A_7]\sigma COS(B-\sigma) = B_8\sigma COS(B-\sigma)$, $[3A_7]\sigma^2 SIN(B-\sigma) = B_9\sigma^2 SIN(B-\sigma)$, $[A_6]\sigma^2 COS(B-\sigma) = B_{10}\sigma^2 COS(B-\sigma)$,
$[A_7]\sigma^3 COS(B-\sigma) = B_{11}\sigma^3 COS(B-\sigma)$, $[1/4\; A_8 - 3/8\; A_{10}]COS2(B-\sigma) = B_{12} COS2(B-\sigma)$,
$[1/4\; A_9 - 3/4\; A_{11}]SIN2(B-\sigma) = B_{13} SIN2(B-\sigma)$, $[-A_8/2 + 3/4\; A_{10}]\sigma SIN2(B-\sigma) = B_{14}\sigma SIN2(B-\sigma)$,
$[1/2\; A_9 - 3/2\; A_{11}]\sigma COS2(B-\sigma) = B_{15}\sigma COS2(B-\sigma)$, $[-1/2\; A_9 + 3/2\; A_{11}]\sigma^2 SIN2(B-\sigma) = B_{16}\sigma^2 SIN2(B-\sigma)$,
$[3/4\; A_{10}]\sigma^2 COS2(B-\sigma) = B_{17}\sigma^2 COS2(B-\sigma)$, $[-1/2\; A_{10}]\sigma^3 SIN2(B-\sigma) = B_{18}\sigma^3 SIN2(B-\sigma)$,
$[A_{11}]\sigma^3 COS2(B-\sigma) = B_{19}\sigma^3 COS2(B-\sigma)$, $[-1/2\; A_{11}]\sigma^4 SIN2(B-\sigma) = B_{20}\sigma^4 SIN2(B-\sigma)$.

**The general solution becomes:**

**37$_c$)**
$$_{VCAV}\mathbf{CENT_Y} = \int {}^{*}[B_1\sigma^2 + B_2\sigma^3 + B_3\sigma^4 + B_4\sigma^5 + B_5 SIN(B-\sigma) + B_6 COS(B-\sigma) + B_7\sigma SIN(B-\sigma)]\; COS\theta\, d\theta$$
$$\int {}^{*} + [B_8\sigma COS(B-\sigma) + B_9\sigma^2 SIN(B-\sigma) + B_{10}\sigma^2 COS(B-\sigma) + B_{11}\sigma^3 COS(B-\sigma) + B_{12} COS2(B-\sigma)]\,COS\theta d\theta$$
$$\int {}^{*} + [B_{13} SIN2(B-\sigma) + B_{14}\sigma SIN2(B-\sigma) + B_{15}\sigma COS2(B-\sigma) + B_{16}\sigma^2 SIN2(B-\sigma)]\; COS\theta\, d\theta$$
$$\int {}^{*}{}_0^{\phi_i} + [B_{17}\sigma^2 COS2(B-\sigma) + B_{18}\sigma^3 SIN2(B-\sigma) + B_{19}\sigma^3 COS2(B-\sigma) + B_{20}\sigma^4 SIN2(B-\sigma)]\; COS\theta\, d\theta.$$

**37$_d$) At the upper limit $\sigma = \phi_i$:**

$$_{VCAV}\mathbf{CENT_Y} = \int[B_1\phi_i^2 + B_2\phi_i^3 + B_3\phi_i^4 + B_4\phi_i^5 + B_5 SING + B_6 COSG + B_7\phi_i SING + B_8\phi_i COSG + B_9\phi_i^2 SING]\,COS\theta d\theta$$
$$+ \int[B_{10}\phi_i^2 COSG + B_{11}\phi_i^3 COSG + B_{12} COS2G + B_{13} SIN2G + B_{14}\phi_i SIN2G + B_{15}\phi_i COS2G]\,COS\theta\, d\theta$$
$$+ \int[B_{16}\phi_i^2 SIN2G + B_{17}\phi_i^2 COS2G + B_{18}\phi_i^3 SIN2G + B_{19}\phi_i^3 COS2G + B_{20}\phi_i^4 SIN2G]\; COS\theta\, d\theta.$$

**37$_e$) At the lower limit $\sigma = 0$:**

$$_{VCAV}\mathbf{CENT_Y} = \int[B_5 SIN(G + \phi_i) + B_6 COS(G + \phi_i) + B_{12} COS\,2(G + \phi_i) + B_{13} SIN\,2(G + \phi_i)]\,COS\theta\, d\theta.$$

**Collecting common coefficients in $37_d$ for,**

$$[B_5\mathit{SING} + B_6\mathit{COSG} + B_{12}\mathit{COS2G} + B_{13}\mathit{SIN2G}] = C_0, \qquad [B_2 + B_{11}\mathit{COSG} + B_{18}\mathit{SIN2G} + B_{19}\mathit{COS2G}]\phi^3 = C_3\phi^3,$$

$$[B_7\mathit{SING} + B_8\mathit{COSG} + B_{14}\mathit{SIN2G} + B_{15}\mathit{COS2G}]\phi = C_1\phi, \qquad [B_3 + B_{20}\mathit{SIN2G}]\phi^4 = C_4\phi^4,$$

$$[B_1 + B_9\mathit{SING} + B_{10}\mathit{COSG} + B_{16}\mathit{SIN2G} + B_{17}\mathit{COS2G}]\phi^2 = C_2\phi^2, \qquad [B_4]\phi^5 = C_5\phi^5.$$

**Equations 37 d and 37 e are now:**

$$37_f) \quad {}_{VCAV}CENT_Y = \int[C_0 + C_1\phi + C_2\phi^2 + C_3\phi^3 + C_4\phi^4 + C_5\phi^5]\mathit{COS}\theta d\theta$$

$$+ \int[-B_5\mathit{SIN}(G + \phi_i) - B_6\mathit{COS}(G + \phi_i) - B_{12}\mathit{COS}2(G + \phi_i) - B_{13}\mathit{SIN}2(G + \phi_i)]\mathit{COS}\theta d\theta.$$

**With $X = G + \alpha_1 + \alpha_2/2$, $L = \alpha_1/1 + \alpha_2/2$, $K = -\alpha_2/2$, $Y = -\alpha_2/2\mathit{COS}u(\theta - ß)$ and recalling $G = A(7)$ is the face surface angle, more extensive expansions for $\mathit{SIN}Y$ and $\mathit{COS}Y$ will be used here then those used in the main bearing load section.**

$\mathit{SIN}(Y) \approx Y - Y^3/6 + Y^5/120 - Y^7/5040$ **and** $\mathit{COS}(Y) = 1 - Y^2/2 + Y^4/24 - Y^6/720.$

$$-B_5\mathit{SIN}(G + \phi_i) = -B_5\mathit{SIN}X\mathit{COS}Y - B_5\mathit{SIN}Y\mathit{COS}X, \qquad -B_{13}\mathit{SIN}2(G + \phi_i) = -B_{13}\mathit{SIN}(2X)\mathit{COS}(2Y) - B_{13}\mathit{COS}(2X)\mathit{SIN}(2Y),$$

$$-B_6\mathit{COS}(G + \phi_i) = -B_6\mathit{COS}X\mathit{COS}Y + B_6\mathit{SIN}X\mathit{SIN}Y, \qquad -B_{12}\mathit{COS}2(G + \phi_i) = -B_{12}\mathit{COS}(2X)\mathit{COS}(2Y) + B_{12}\mathit{SIN}(2X)\mathit{SIN}(2Y)$$

**For clutter control:**

$$[-B_5\mathit{SIN}(X) - B_6\mathit{COS}(X)]\mathit{COS}(Y) = D_1[1 - Y^2/2 + Y^4/24 - Y^6/720],$$

$$[-B_5\mathit{COS}(X) + B_6\mathit{SIN}(X)]\mathit{SIN}(Y) = D_2[Y - Y^3/6 + Y^5/120 - Y^7/5040],$$

$$[-B_{13}\mathit{SIN}(2X) - B_{12}\mathit{COS}(2X)]\mathit{COS}(2Y) = D_3[1 - 2Y^2 + 2Y^4/3 - 4Y^6/45],$$

$$[-B_{13}\mathit{COS}(2X) + B_{12}\mathit{SIN}(2X)]\mathit{SIN}(2Y) = D_4[2Y - 4Y^3/3 + 4Y^5/15 - 8Y^7/315].$$

**Using partition angle $\phi_i = \alpha_1 + \alpha_2/2[1 - \mathit{COS}u(\theta - ß)] = L + Y$,**

$$C_1\phi_i = C_1(L + Y), \qquad C_2\phi_i^2 = C_2(L^2 + 2LY + Y^2), \qquad C_3\phi_i^3 = C_3(L^3 + 3L^2Y + 3LY^2 + Y^3)$$

$$C_4\phi_i^4 = C_4(L^4 + 4L^3Y + 6L^2Y^2 + 4LY^3 + Y^4), \qquad C_5\phi_i^5 = C_5(L^5 + 5L^4Y + 10L^3Y^2 + 10L^2Y^3 + 5LY^4 + Y^5).$$

**Collecting common coefficients:**

$$[C_0 + C_1L + C_2L^2 + C_3L^3 + C_4L^4 + C_5L^5 + D_1 + D_3] = E_0,$$

$$[C_1 + 2C_2L + 3C_3L^2 + 4C_4L^3 + 5C_5L^4 + D_{2J} + 2D_4](-\alpha_2/2)\mathit{COS}u(\theta - ß) = E_1\mathit{COS}u(\theta - ß),$$

$$[C_2 + 3C_3L + 6C_4L^2 + 10C_5L^3 - D_1/2 - 2D_3](-\alpha_2/2)^2\mathit{COS}^2u(\theta - ß) = E_2\mathit{COS}^2u(\theta - ß),$$

$$[C_3 + 4C_4L + 10C_5L^2 - D_2/6 - 4D_4/3](-\alpha_2/2)^3\mathit{COS}^3u(\theta - ß) = E_3\mathit{COS}^3u(\theta - ß),$$

$$[C_4 + 5C_5L + D_1/24 + 2D_3/3](-\alpha_2/2)^4\mathit{COS}^4u(\theta - ß) = E_4\mathit{COS}^4u(\theta - ß),$$

$$[C_5 + D_2/120 + 4D_4/15](-\alpha_2/2)^5\mathit{COS}^5u(\theta - ß) = E_5\mathit{COS}^5u(\theta - ß),$$

$$[-D_1/720 - 4D_3/45](-\alpha_2/2)^6\mathit{COS}^6u(\theta - ß) = E_6\mathit{COS}^6u(\theta - ß),$$

$$[-D_2/5040 - 8D_4/315](-\alpha_2/2)^7\mathit{COS}^7u(\theta - ß) = E_7\mathit{COS}^7u(\theta - ß).$$

**The integral below is the first moment of the annular cavity variable region about the rotor coordinate Y-axis.**

$$37_g) \quad {}_{VCAV}CENT_Y = \int[E_0 + E_1\mathit{COS}u(\theta - ß) + E_2\mathit{COS}^2u(\theta - ß) + E_3\mathit{COS}^3u(\theta - ß)]\mathit{COS}\theta\, d\theta$$

$$+ \int[E_4\mathit{COS}^4u(\theta - ß) + E_5\mathit{COS}^5u(\theta - ß) + E_6\mathit{COS}^6u(\theta - ß) + E_7\mathit{COS}^7u(\theta - ß)]\mathit{COS}\theta\, d\theta.$$

**Using the math appendix 67 - 73 solutions of $\int COS^n u(\theta - ß)\mathit{COS}\theta\, d\theta$ for terms in 37 g above with,**

$$r_0 = 1/u,\ r_1 = 1/u^2, \quad r_2 = 1/(u^2 - 1), \quad r_3 = 1/(1 - 9u^2), \quad r_4 = 1/(1 - 16u^2), \quad r_5 = 1/(1 - 25u^2),$$

$$r_6 = 1/(1 - 36u^2), \quad r_7 = 1/(1 - 49u_2), \quad S_0 = 1/(2u), \quad s_1 = 1/(4u^2), \text{ and } \quad s_2 = 1/(4u^2 - 1),$$

$$E_0\int \mathit{COS}\theta\, d\theta = E_0\mathit{SIN}\theta. \qquad E_1\int \mathit{COS}u\theta\mathit{COS}\theta\, d\theta = -r_2E_1[\mathit{COS}u\theta\mathit{SIN}\theta - u\mathit{SIN}u\theta\mathit{COS}\theta].$$

$$E_2\int \mathit{COS}^2u\theta\mathit{COS}\theta\, d\theta = E_2/2\ \mathit{SIN}\theta - 1/2\ E_2s_2[\mathit{COS}2u\theta\mathit{SIN}\theta - 2u\mathit{SIN}2u\theta\mathit{COS}\theta].$$

$$E_3\int \mathit{COS}^3u\theta\mathit{COS}\theta\, d\theta = r_3E_3[\mathit{COS}^3u\theta\mathit{SIN}\theta - 3u\mathit{COS}^2u\theta\mathit{SIN}u\theta\mathit{COS}\theta] + 6E_3u^4r_3r_2[\mathit{COS}u\theta\mathit{SIN}\theta - \mathit{SIN}u\theta\mathit{COS}\theta].$$

$$E_4\int \mathit{COS}^4u\theta\mathit{COS}\theta d\theta = E_4r_4[\mathit{COS}^4u\theta\mathit{SIN}\theta - 4u\mathit{COS}^3u\theta\mathit{SIN}u\theta\mathit{COS}\theta - 6u^2\mathit{SIN}\theta] + 6E_4u^2r_4s_2[\mathit{COS}2u\theta\mathit{SIN}\theta - 2u\mathit{SIN}2u\theta\mathit{COS}\theta].$$

$$E_5\int \mathit{COS}^5u\theta\mathit{COS}\theta\, d\theta = E_5r_5[\mathit{COS}^5u\theta\mathit{SIN}\theta - 5u\mathit{COS}^4u\theta\mathit{SIN}u\theta\mathit{COS}\theta] - 20E_5u^2r_3r_5[\mathit{COS}^3u\theta\mathit{SIN}\theta - 3u\mathit{COS}^2u\theta\mathit{SIN}u\theta\mathit{COS}\theta]$$

$$- 120E_5u^6r_3r_5r_2[\mathit{COS}u\theta\mathit{SIN}\theta - u\mathit{SIN}u\theta\mathit{COS}\theta].$$

$$E_6\int \mathit{COS}^6u\theta\mathit{COS}\theta\, d\theta = E_6r_6[\mathit{COS}^6u\theta\mathit{SIN}\theta - 6u\mathit{SIN}u\theta\mathit{COS}^5u\theta\mathit{COS}\theta]$$

$$- 30E_6u^2r_6r_4[\mathit{COS}^4u\theta\mathit{SIN}\theta - 4u\mathit{COS}^3u\theta\mathit{SIN}u\theta\mathit{COS}\theta - 6u^2\mathit{SIN}\theta] - 180E_6u^4r_6r_4s_2[\mathit{COS}2u\theta\mathit{SIN}\theta - 2u\mathit{SIN}2u\theta\mathit{COS}\theta].$$

$$E_7\int \mathit{COS}^7u\theta\mathit{COS}\theta\, d\theta = E_7r_7\mathit{COS}^7u\theta\mathit{SIN}\theta - 7E_7ur_7\mathit{COS}^6u\theta\mathit{SIN}u\theta\mathit{COS}\theta - 42E_7u^2r_7r_5[\mathit{COS}^5u\theta\mathit{SIN}\theta - 5u\mathit{COS}^4u\theta\mathit{SIN}u\theta\mathit{COS}\theta]$$

$$+ 840E_7u^4r_7r_5r_3[\mathit{COS}^3u\theta\mathit{SIN}\theta - 3u\mathit{COS}^2u\theta\mathit{SIN}u\theta\mathit{COS}\theta] + 5040E_7u^8r^7r_5r_3r_2[\mathit{COS}u\theta\mathit{SIN}\theta - u\mathit{SIN}u\theta\mathit{COS}\theta].$$

**Collecting the coefficients of common terms:**

$$A_{my0}\mathit{SIN}\theta = [E_0 + E_2/2 - 6u^2E_4r_4 + 180E_6u^4r_6r_4]\mathit{SIN}\theta,$$

$$A_{my1}\mathit{COS}u(\theta - ß)\mathit{SIN}\theta = [-r_2E_1 + 6E_3u^2r_3r_2 - 120E_5u^6r_5r_3r_2 + 5040E_7u^7r_7r_5r_3r_2]\mathit{COS}u(\theta - ß)\mathit{SIN}\theta,$$

$$A_{my2}\mathit{SIN}u(\theta - ß)\mathit{COS}\theta = [r_2E_1u - 6E_3u^5r_3r_2 + 120E_5u^7r_5r_3r_2 - 5040E_7u^7r_7r_5r_3r_2]\mathit{SIN}u(\theta - ß)\mathit{COS}\theta,$$

$$A_{my3}\mathit{COS}2u(\theta - ß)\mathit{SIN}\theta = [-1/2E_2s_2 + 6E_4u^2r_4s_2 - 180E_6u^4r_6r_4s_2]\mathit{COS}2u(\theta - ß)\mathit{SIN}\theta,$$

$$A_{my4}\,SIN2u(\theta-\beta)COS\theta = [E_2s_2u - 12E_4u^3r_4s_2 + 360E_6u^5r_6r_4s_2]\,SIN2u(\theta-\beta)COS\theta,$$

$$A_{my5}COS^2u(\theta-\beta)SINu(\theta-\beta)COS\theta = [-3ur_3E_3 + 60E_5u^3r_5r_3 - 2520E_7u^5r_7r_5r_3]\,COS^2u(\theta-\beta)SINu(\theta-\beta)COS\theta,$$

$$A_{my6}COS^3u(\theta-\beta)SINu(\theta-\beta)COS\theta = [-4uE_4r_4 + 120E_6u^3r_6r_4]\,COS^3u(\theta-\beta)SINu(\theta-\beta)COS\theta,$$

$$A_{my7}COS^4u(\theta-\beta)SINu(\theta-\beta)COS\theta = [-5uE_5r_5 + 210E_7u^3r_7r_5]\,COS^4u(\theta-\beta)SINu(\theta-\beta)COS\theta,$$

$$A_{my8}COS^5u(\theta-\beta)SINu(\theta-\beta)COS\theta = [-6uE_6r_6]\,COS^5u(\theta-\beta)SINu(\theta-\beta)COS\theta,$$

$$A_{my9}COS^6u(\theta-\beta)SINu(\theta-\beta)COS\theta = [-7uE_7r_7]\,COS^6u(\theta-\beta)SINu(\theta-\beta)COS\theta,$$

$$A_{my10}COS^3u(\theta-\beta)SIN\theta = [r_3E_3 - 20E_5u^2r_5r_3 + 840E_7u^4r_7r_5r_3]COS^3u(\theta-\beta)SIN\theta,$$

$$A_{my11}COS^4u(\theta-\beta)SIN\theta = [E_4r_4 - 30E_6u^2r_6r_4]\,COS^4u(\theta-\beta)SIN\theta, \quad A_{my12}COS^5u(\theta-\beta)SIN\theta = [E_5r_5 - 42E_7u^2r_7r_5]COS^5u(\theta-\beta)SIN\theta,$$

$$A_{my13}COS^6u(\theta-\beta)SIN\theta = [E_6r_6]\,COS^6u(\theta-\beta)SIN\theta, \quad A_{my14}COS^7u(\theta-\beta)SIN\theta = [E_7r_7]\,COS^7u(\theta-\beta)SIN\theta.$$

**An annular cavity variable volume segment between $\theta_1$ and $\theta_2$ has the Y-axis moment:**

$$37_h)\quad {}_{VCAV}CENT_Y = A_{my0}SIN\theta + A_{my1}COSu(\theta-\beta)SIN\theta + A_{my2}SINu(\theta-\beta)COS\theta + Au_{my3}COS2u(\theta-\beta)SIN\theta$$
$$+ A_{my4}SIN2u(\theta-\beta)COS\theta + A_{my5}COS^2u(\theta-\beta)SINu(\theta-\beta)COS\theta + A_{my6}COS^3u(\theta-\beta)SINu(\theta-\beta)COS\theta$$
$$+ A_{my7}COS^4u(\theta-\beta)SINu(\theta-\beta)COS\theta + A_{my8}COS^5u(\theta-\beta)SINu(\theta-\beta)COS\theta + A_{my9}COS^6u(\theta-\beta)SINu(\theta-\beta)COS\theta$$
$$+ A_{my10}COS^3u(\theta-\beta)SIN\theta + A_{my11}COS^4u(\theta-\beta)SIN\theta + A_{my12}COS^5u(\theta-\beta)SIN\theta + A_{MY13}COS^6u(\theta-\beta)SIN\theta + A_{MY14}COS^7u(\theta-\beta)SIN\theta.$$

## 38 ANNULAR CAVITY'S Y-AXIS MOMENTS (CONSTANT PART).

**Equation 37 above is utilized to find the annular cavity's minimum and maximum volume regions Y-axis moment.**

$$38)\quad {}_{VCAV}CENT_Y = \int_{\theta_1}^{\theta_2}\int_0^{\phi_c}\int_a^{r=a+N\sigma} \Big[\underbrace{[R + rSIN(B-\sigma)]COS\theta}_{\{MOMENT\ ARM\}}\Big]\Big[\underbrace{[R + rSIN(B-\sigma)]\,d\theta\, r\, d\sigma\, dr}_{\{VOLUME\ ELEMENT,\ dA\}}\Big].$$

**Operations towards a solution of 38 are identical to those for equation 37c, but $\phi_c$ has at constant value $\alpha_1$ in the cavity's minimum volume regions and value $\alpha_1 + \alpha_2$ in the cavity's maximum volume regions and the solution is:**

$38_a)$

$$ {}_{VCAV}CENT_Y = \int_0^{\phi_i} {}^{*}[B_1\sigma^2 + B_2\sigma^3 + B_3\sigma^4 + B_4\sigma^5 + B_5SIN(B-\sigma) + B_6COS(B-\sigma)]\,COS\theta\, d\theta$$
$$+ \int^{*}[B_7\sigma SIN(B-\sigma) + B_8\sigma COS(B-\sigma) + B_9\sigma^2SIN(B-\sigma) + B_{10}\sigma^2COS(B-\sigma) + B_{11}COS2(B-\sigma)]COS\theta d\theta$$
$$+ \int^{*}[B_{12}SIN2(B-\sigma) + B_{13}\sigma SIN2(B-\sigma) + B_{14}\sigma COS2(B-\sigma) + B_{15}\sigma^2SIN2(B-\sigma)]COS\theta d\theta$$
$$+ \int^{*}[B_{16}\sigma^2COS2(B-\sigma) + B_{17}\sigma^3SIN2(B-\sigma) + B_{18}\sigma^3COS2(B-\sigma) + B_{19}\sigma^4SIN2(B-\sigma)]\,COS\theta\, d\theta.$$

**In the cavity's largest volume region the upper limit is $\sigma = \phi_c = \alpha_1 + \alpha_2 = A(12)$ and**

$$38_b)\quad {}_{VCAV}CENT_Y = \int[B_1\phi_c^2 + B_2\phi_c^3 + B_3\phi_c^4 + B_4\phi_c^5 + B_5SING + B_6COSG]\,COS\theta\, d\theta$$
$$+ \int[B_7\phi_cSING + B_8\phi_cCOSG + B_9\phi_c^2SING + B_{10}\phi_c^2COSG]\,COS\theta\, d\theta$$
$$+ \int[B_{11}COS2G + B_{12}SIN2G + B_{13}\phi_cSIN2G + B_{14}\phi_cCOS2G + B_{15}\phi_c^2SIN2G]\,COS\theta\, d\theta$$
$$+ \int[B_{16}\phi_c^2COS2G + B_{17}\phi_c^3SIN2G + B_{18}\phi_c^3COS2G + B_{19}\phi_c^4SIN2G]\,COS\theta\, d\theta.$$

**$38_c)$ At the lower limit $\sigma = 0$ and with $H = (G + A(12))$**

$$ {}_{VCAV}CENT_Y = \int[-B_5SINH - B_6COSH - B_{11}COS2H - B_{12}SIN2H]\,COS\theta\, d\theta$$

**A maximum volume annular cavity region has Y-axis centroid between $\theta_2$ and $\theta_1$:**

$$38_d)\quad {}_{MXVOL}CENT_Y = [B_1(\alpha_1+\alpha_2)^2 + B_2(\alpha_1+\alpha_2)^3 + B_3(\alpha_1+\alpha_2)^4 + B_4(\alpha_1+\alpha_2)^5 + B_7(\alpha_1+\alpha_2)SING][SIN\theta_2 - SIN\theta_1]$$
$$+ [B_8(\alpha_1+\alpha_2)COSG + B_9(\alpha_1+\alpha_2)^2SING + B_{10}(\alpha_1+\alpha_2)^2COSG + B_{11}COS2G][SIN\theta_2 - SIN\theta_1]$$
$$+ [B_{12}SIN2G + B_{13}(\alpha_1+\alpha_2)SIN2G + B_{14}(\alpha_1+\alpha_2)COS2G + B_{15}((\alpha_1+\alpha_2))^2SIN2G][SIN\theta_2 - SIN\theta_1]$$
$$+ [B_{16}((\alpha_1+\alpha_2))^2COS2G + B_{17}((\alpha_1+\alpha_2))^3SIN2G + B_{18}((\alpha_1+\alpha_2))^3COS2G)][SIN\theta_2 - SIN\theta_1]$$
$$+ [B_{19}((\alpha_1+\alpha_2))^4SIN2G) + B_5SING + B_6COSG + B_{11}COS2G + B_{12}SIN2G][SIN\theta_2 - SIN\theta_1]$$
$$- [B_5SIN(G+\alpha_1+\alpha_2) + B_6COS(G+\alpha_1+\alpha_2) + B_{11}COS2(G+\alpha_1+\alpha_2) + B_{12}SIN2(G+\alpha_1+\alpha_2)][SIN\theta_2 - SIN\theta_1].$$

**A minimum volume cavity region where $\phi_c = \alpha_1 = A(5)$ and $I = G + \alpha_1$, has Y-axis centroid:**

$$38_e)\quad {}_{MNVOL}CENT_Y = [B_1\alpha_1^2 + B_2\alpha_1^3 + B_3\alpha_1^4 + B_4\alpha_1^5 + B_7\alpha_1SING + B_8\alpha_1COS\,G + B_9\alpha_1^2SING][SIN\theta_2 - SIN\theta_1]$$
$$+ [B_{10}\alpha_1^2COSG + B_{11}COS2G + B_{12}SIN2G + B_{13}\alpha_1SIN2G + B_{14}\alpha_1COS2G][SIN\theta_2 - SIN\theta_1]$$
$$+ [B_{15}\alpha_1^2SIN2G + B_{16}\alpha_1^2COS2G + B_{17}\alpha_1^3SIN2G + B_{18}\alpha_1^3COS2G][SIN\theta_2 - SIN\theta_1]$$
$$+ [-\alpha_1^4SIN2G + B_5SING + B_6COSG + B_{11}COS2G + B_{12}SIN2G][SIN\theta_2 - SIN\theta_1]$$
$$- [B_5SIN(G+\alpha_1) + B_6COS(G+\alpha_1) + B_{11}COS(2(G+\alpha_1)) + B_{12}SIN(2(G+\alpha_1)I)][SIN\theta_2 - SIN\theta_1]$$

## 39 ANNULAR CAVITY'S X-AXIS MOMENT (VARYING PART).

**Usually symmetric about the X-axis, the cavity's X-axis moment is rarely needed; however, it's included here.**

$$39)\quad {}_{VCAV}CENT_Y = \int_{\theta_1}^{\theta_2}\int_{0}^{\phi_c}\int_{a}^{r=a+N\sigma}\overbrace{[[R + rSIN(B - \sigma)]SIN\theta]}^{\{MOMENT\ ARM\}}\overbrace{[R + rSIN(B - \sigma)]\, d\theta\, r\, d\sigma\, dr}^{\{VOLUME\ ELEMENT,\ dA\}}$$

**The solutions and constants generated in section 37 for the first two integrals above, $\int dr$ and $\int d\sigma$, are identical to those required in 39. Using the constants generated in section 37, we proceed directly to the solution of 39a below for the annular cavity's varying volume segment moment about the X-axis.**

$$39_a)\quad {}_{VCAV}CENT_X = \int[E_0 + E_1COS\,u(\theta - ß) + E_2COS^2u(\theta - ß) + E_3COS^3u(\theta - ß)]SIN\theta\, d\theta$$
$$+ \int[E_4COS^4u(\theta - ß) + E_5COS^5u(\theta - ß)E_6COS^6u(\theta - ß) + E_7COS^7u(\theta - ß)]SIN\theta\, d\theta.$$

**The math appendix solutions for the elements of the above integrand are:**

$E_0\int SIN\theta\, d\theta = -E_0COS\theta$, $\quad E_1\int COSu\theta SIN\theta\, d\theta = E_1r_2[uSINu\theta SIN\theta + COSu\theta COS\theta]$.

$E_2\int COS^2u\theta SIN\theta\, d\theta = -1/2E_2COS\theta + E_2s_2[uSIN2u\theta SIN\theta + (1/2)COS2u\theta COS\theta]$.

$E_3\int COS^3u\theta SIN\theta\, d\theta = -E_3r_3[COS^3u\theta COS\theta + 3uCOS^2u\theta SINu\theta SIN\theta] - 6E_3r_3r_2[u^3SINu\theta SIN\theta + u^2COSu\theta COS\theta]$.

$E_4\int COS^4u\theta SIN\theta\, d\theta = E_4r_4[-COS^4u\theta COS\theta - 4uCOS^3u\theta SINu\theta SIN\theta + 6u^2COS\theta] - E_4r_4s_2[12u^3SIN2u\theta SIN\theta + 6u^2COS2u\theta COS\theta]$.

$E_5\int COS^5u\theta SIN\theta\, d\theta = E_5r_5[-COS^5u\theta COS\theta - 5uCOS^4u\theta SINu\theta SIN\theta] + 20E_5u^2r_5r_3[COS^3u\theta COS\theta + 3uCOS^2u\theta SINu\theta SIN\theta]$
$\quad + 120E_5u^6r_5r_3r_2[1/uSINu\theta SIN\theta + 1/u^2COSu\theta COS\theta]$.

$E_6\int COS^6u\theta\, SIN\theta d\theta = E_6r_6[-COS\theta COS^6u\theta - 6uCOS^5u\theta SINu\theta SIN\theta]$
$\quad - 30E_6u^2r_6r_4[-COS^4u\theta COS\theta - 4uCOS^3u\theta SINu\theta SIN\theta + 6u^2COS\theta]$
$\quad + 720E_6u^6r_6r_4s_2[1/2uSIN2u\theta SIN\theta + 1/(4u^2)COS2u\theta COS\theta]$.

$E_7\int COS^7u\theta SIN\theta d\theta = -E_7r_7COS^7u\theta COS\theta - 7E_7ur_7COS^6u\theta SINu\theta SIN\theta - 42E_7u^2r_7r_5[-COS^5u\theta COS\theta - 5uCOS^4u\theta SINu\theta SIN\theta]$
$\quad - 840E_7u^4r_7r_5r_3[COS^3u\theta COS\theta + 3uCOS^2u\theta SINu\theta SIN\theta] - 5040E_7u^6r_7r_5r_3r_2[uSINu\theta SIN\theta + COSu\theta COS\theta]$.

**Collecting coefficients of common terms:**

$A_{mx0}COS\theta = [-E_0 - E_2/2 + 6u^2E_4r_4 - 180u^4E_6r_6r_4]COS\theta$,

$A_{mx1}COSu(\theta - ß)COS\theta = [E_1r_2 - 6u^2E_3r_3r_2 + 120u^4E_5r_5r_3r_2 - 5040u^6E_7r_7r_5r_3r_2]COSu(\theta - ß)COS\theta$

$A_{mx2}SINu(\theta - ß)SIN\theta = [uE_1r_2 - 6u^3E_3r_3r_2 + 120u^5E_5r_5r_3r_2 - 5040u^7E_7r_7r_5r_3r_2]SINu(\theta - ß)SIN\theta$,

$A_{mx3}COS2u(\theta - ß)COS\theta = [E_2s_2/2 - 6u^2E_4r_4s_2 + 180u^4E_6r_6r_4s_2]COS2u(\theta - ß)COS\theta$

$A_{mx4}SIN2u(\theta - ß)SIN\theta = [uE_2s_2 - 12u^3E_4r_4s_2 + 360u^5E_6r_6r_4s_2]SIN2u(\theta - ß)SIN\theta$

$A_{mx5}COS^2u(\theta - ß)SINu(\theta - ß)SIN\theta = [-3uE_3r_3 + 60u^3E_5r_5r_3 - 2520u^5E_7r_7r_5r_3]COS^2u(\theta - ß)SINu(\theta - ß)SIN\theta$

$A_{mx6}COS^3u(\theta - ß)SINu(\theta - ß)SIN\theta = [-4uE_4r_4 + 120u^3E_6r_6r_4]COS^3u(\theta - ß)SINu(\theta - ß)SIN\theta$,

$A_{mx7}COS^4u(\theta - ß)SINu(\theta - ß)SIN\theta = [-5uE_5r_5 + 210u^3E_7r_7r_5]COS^4u(\theta - ß)SINu(\theta - ß)SIN\theta$,

$A_{mx8}COS^5u(\theta - ß)SINu(\theta - ß)SIN\theta = [-6uE_6r_6]COS^5u(\theta - ß)SINu(\theta - ß)SIN\theta$,

$A_{mx9}COS^6u(\theta - ß)SINu(\theta - ß)SIN\theta = [-7uE_7r_7]COS^6u(\theta - ß)SINu(\theta - ß)SIN\theta$,

$A_{mx10}COS^3u(\theta - ß)COS\theta = [-E_3r_3 + 20u^2E_5r_5r_3 - 840u^4E_7r_7r_5r_3]COS^3u(\theta - ß)COS\theta$,

$A_{mx11}COS^4u(\theta - ß)COS\theta = [-E_4r_4 + 30u^2E_6r_6r_4]COS^4u(\theta - ß)COS\theta$,

$A_{mx12}COS^5u(\theta - ß)COS\theta = [-E_5r_5 + 42u^2E_7r_7r_5]COS^5u(\theta - ß)COS\theta$,

$A_{mx13}COS^6u(\theta - ß)COS\theta = [-E_6r_6]COS^6u(\theta - ß)COS\theta$, $\quad A_{mx14}COS^7u(\theta - ß)COS\theta = [-E_7r_7]COS^7u(\theta - ß)COS\theta$.

**Equation 39c below is now evaluated between the limits $\theta_2$ and $\theta_1$ of the region.**

$$39_c)\quad {}_{VCAV}CENT_X = A_{mx0}COS\theta + A_{mx1}COSu(\theta - ß)COS\theta + A_{mx2}SINu(\theta - ß)SIN\theta + A_{mx3}COS2u(\theta - ß)COS\theta$$
$$+ A_{mx4}SIN2u(\theta - ß)SIN\theta + A_{mx5}COS^2u(\theta - ß)SINu(\theta - ß)SIN\theta + A_{mx6}COS^3u(\theta - ß)SINu(\theta - ß)SIN\theta$$
$$+ A_{mx7}COS^4u(\theta - ß)SINu(\theta - ß)SIN\theta + A_{mx8}COS^5u(\theta - ß)SIN\,u(\theta - ß)SIN\theta + A_{mx9}COS^6u(\theta - ß)SINu(\theta - ß)SIN\theta$$
$$+ A_{mx10}COS^3u(\theta-ß)COS\theta + A_{mx11}COS^4u(\theta-ß)COS\theta + A_{mx12}COS^5u(\theta-ß)COS\theta + A_{mx13}COS^6u(\theta-ß)COS\theta + A_{mx14}COS^7u(\theta-ß)COS\theta.$$

## 40 CONSTANT VOLUME CAVITY REGION'S X-AXIS MOMENT.

**From section 38 the maximum volume region's X-axis moments is:(where $\phi_c = \alpha_1 + \alpha_2$ and $H = G + \alpha_1 + \alpha_2$),**

$$40)\quad {}_{MXVOL}CENT_X = [B_1\phi_c^2 + B_2\phi_c^3 + B_3\phi_c^4 + B_4\phi_c^5 + B_7\phi_cSING + B_8\phi_cCOSG + B_9\phi_c^2SING + B_{10}\phi_c^2COSG][COS\theta_1 - COS\theta_2]$$
$$+ [B_{11}COS2G + B_{12}SIN2G + B_{13}\phi_cSIN2G + B_{14}\phi_cCOS2G + B_{15}\phi_c^2SIN2G + B_{16}\phi_c^2COS2G][COS\theta_1 - COS\theta_2]$$
$$+ [B_{17}\phi_c^3SIN2G + B_{18}\phi_c^3COS2G + B_{19}\phi_c^4SIN2G + B_5SING + B_6COSG + B_{11}COS2G + B_{12}SIN2G]][COS\theta_1 - COS\theta_2]$$
$$- [B_5SINH + B_6COSH + B_{11}COS2H + B_{12}SIN2H][COS\theta_1 - COS\theta_2].$$