

$$16) \quad M = \int_{-t/2}^{t/2} [-\sigma_{mx} [z(t/2)] z \Delta x dz = (-2\sigma_{mx}/t) \Delta x \int_{-t/2}^{t/2} z^2 dz = (-2\sigma_{mx}/t) \Delta x [t^3/8 - (-t^3/8)]/3$$

16_a) $M = \Delta x t^2 \sigma_{mx} / 6$
 Imposing a moment M on a beam element, causes a transverse section of the beam element to experience a maximum stress, σ_{mx} , of,

$$16_b) \quad \sigma_{mx} = 6M / (\Delta x t^2).$$

The maximum elastic stress, σ_{max} , of the beam material limits the maximum moment, M_{max} , the beam can tolerate before permanent deformation. This is expressed as:

$$16_c) \quad M_{max} = (\Delta x t^2 \sigma_{max}) / 6.$$

The stress at distance z from the neutral axis is $\sigma_z = z \sigma_{mx} / (t/2)$ and when the beam element experiences moment, M_{mx} , substitution into 16c yields the stress, σ_z , at any z, $M_{mx} = (\Delta x t^2 [(t/2) \sigma_z / z]) / 6 = (\Delta x t^3 \sigma_z) / (12z)$,

$$16_d) \quad \sigma_z = 12 M_{mx} z / (\Delta x t^3).$$

To determine the beam shear stress, τ , we first refer to figure 16 a, and sum the moments about point P on the beam element as:

$$\sum M_p = \int f(y) dy dy/2 + (M + dM) - (V + dV) dy$$

Disregarding all differential products of degrees higher than one:

$$16_e) \quad dM/dy = V.$$

As the outer surface of the beam has no shear stresses and the shear stress in planes parallel to the outer surface have shear force, $\tau dx dy$, this shear is balanced by the stress forces at the beam section. With reference figure 16 b, a force balance is done of a beam element which is parallel the outer surface. $F_a = F_b + \tau dx dy$. Equation 16 d gives the stress σ_z at z above the neutral axis. Its apparent that the greatest value of τ is at the neutral plane, so:

$$16_f) \quad F_a = \int_0^{t/2} [12 M_a z / (\Delta x t^3)] \Delta x dz = (12 M_a / t^3) (t^2/4) / 2 = 3/2 M_a / t,$$

$$16_g) \quad F_b = \int_0^{t/2} 12 (M_a + dM) z / (\Delta x t^3) \Delta x dz = [12 (M_a + dM) / t^3] (t^2/4) / 2 = 3/2 (M_a + dM) / t.$$

$$16_h) \quad F_a - F_b = \tau \Delta x dy = - 3/2 dM/t.$$

Using 16 e above, $V = dM/dy$, for :

$$16_i) \quad \tau_{max} = - 3/2 (V dy) / (t \Delta x dy) = - 3/2 V / (t \Delta x),$$

which is at the neutral axis.

Partition area Equation 6a is used to find the load, V_p , at ϕ_{01} :

$$16_j) \quad V_p = dP (a N \phi_{01}^2 / 2 + N^2 \phi_{01}^3 / 6) \quad (Fn Shear)$$

Using the polar moment equation:

$$M_p = \Delta p \int_0^{\phi_{01}} \int_a^{a+N\delta 1} r^2 d\delta dr$$

$$16_k) \quad M_p \leq dP (a^2 N \phi_{01} + a N^2 \phi_{01}^2 + N^3 \phi_{01}^3 / 3) \quad (Fn M polar)$$

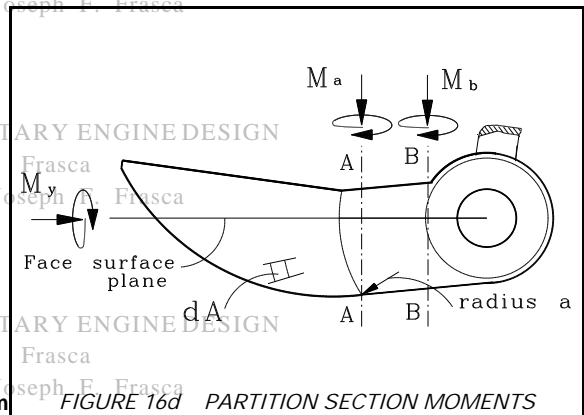
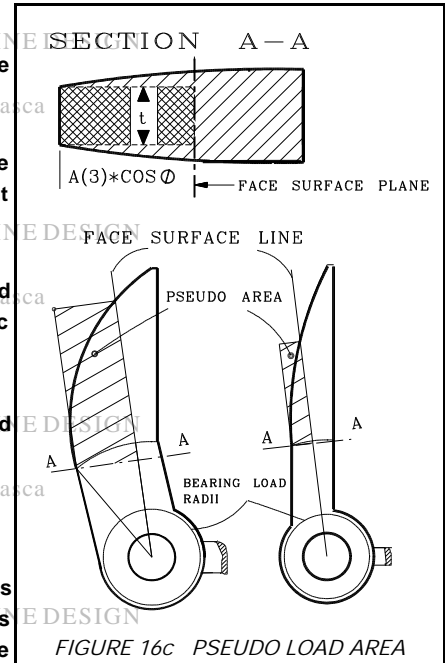
To maximize the design parameters while keeping them

reasonable for engine design, the computer program utilizes the above polar moment and the following pseudo area and its moment in engine design. Using the program supplied partition loading values at the various angles, ϕ_{01} with commercial FEA programming, the design engineer should be able to reduce significantly the partition mass required by the engine design the program.

With reference to figure 16 c, the pseudo area is indicated by hatching and the section thickness utilized by the program as the partition thickness at the minimum radius, A(3), is indicated by the cross hatching in the section A - A. The excess thickness shown, is due to the partition's radial taper - discussed later - to decrease mass.

With the partition's maximum radius, $y_m = a + N \phi_{01}$, (at the face surface) and minimum radius $y_s = a \cos \phi_{01}$, the pseudo area width is: $x_s = a \sin \phi_{01}$, and pseudo area is:

$$16_l) \quad Area_{pseudo} = [(a + N \phi_{01}) - a \cos \phi_{01}] (a \sin \phi_{01}). \quad (Fn Apseudo)$$



The pseudo area moment is taken at r_{bx} , where $r_{bx} = r_b \cos(\alpha_1 + \alpha_2)$, and r_b is the pivot bearing load radius.

$$16_m) \quad M_{B-B}^{pseuso} = dP \int_{y_s - r_{bx}}^{y_m - r_{bx}} x_s y dy = dP \int_{y_s - r_{bx}}^{y_m - r_{bx}} x_s y^2/2 = dP x_s [(y_m^2 - 2y_m r_{bx} + r_{bx}^2) - (y_s^2 - 2y_s r_{bx} + r_{bx}^2)]/2$$

$$M_{B-B}^{pseuso} = dP x_s [(y_m^2 - y_s^2)/2 - (y_m - y_s) r_{bx}] = [dP x_s (y_m - y_s)/2][(y_m + y_s) - 2r_{bx}] \quad (F_n M_{bbp})$$

The moment of the pseudo area about a section taken at radius $a \cos \phi_{\theta}$ is:

$$16_n) \quad M_{A-A}^{pseuso} = dP \int_0^{y_m - y_s} x_s y dy = dP \int_0^{y_m - y_s} x_s y^2/2 = [dP x_s (y_m - y_s)/2](y_m - y_s) \quad (F_n M_{aap})$$

Moments of the partition area at the sections in figure 16d are:

$$16_o) \quad M_{A-A} = dP \int_0^{\phi_{\theta}} \int_a^{a+N\delta} r [r \cos(\phi_{\theta} - \delta) - a \cos \phi_{\theta}] d\delta dr,$$

$$16_o1) \quad M_{A-A} = dP \int_0^{\phi_{\theta}} [a^2 N \delta + a N^2 \delta^2 + N^3 \delta^3/3] \cos(\phi - \delta) - [a^2 N \delta + a N^2 \delta^2/2] \cos \phi d\delta.$$

Utilizing the math appendix 44 - 48 for solutions of $\int \delta^n \cos(\phi - \delta) d\delta$:

$$\int a^2 N \delta \cos(\phi - \delta) d\delta = a^2 N [-\delta \sin(\phi - \delta) + \cos(\phi - \delta)],$$

$$\int a N^2 \delta^2 \cos(\phi - \delta) d\delta = a N^2 [-\delta^2 \sin(\phi - \delta) + 2\delta \cos(\phi - \delta) + 2 \sin(\phi - \delta)],$$

$$\int N^3/3 \delta^3 \cos(\phi - \delta) d\delta = N^3/3 [-\delta^3 \sin(\phi - \delta) + 3\delta^2 \cos(\phi - \delta) + 6\delta \sin(\phi - \delta) - 6 \cos(\phi - \delta)],$$

and collecting common terms in δ for:

$$[-N^3/3] \delta^3 \sin(\phi - \delta), \quad [2a^2 N^2] \delta^2 \sin(\phi - \delta), \quad [2N^3 - a^2 N] \delta \sin(\phi - \delta), \quad [2a^2 N^2] \sin(\phi - \delta)$$

$$[N^3] \delta^2 \cos(\phi - \delta), \quad [2a^2 N] \delta \cos(\phi - \delta), \quad [a^2 N - 2N^3] \cos(\phi - \delta), \quad [-a^2 N \delta^2/2 - a N^2 \delta^3/6] \cos \phi,$$

equation 16o1 evaluated at limits $\delta = \phi_{\theta}$ and $\delta = 0$ is:

$$16_o2) \quad M_{A-A} = dP [-2a^2 N^2] \sin \phi + [2N^3 - a^2 N - a^2 N \delta^2/2 - a N^2 \delta^3/6] \cos \phi + [N^3 \phi^2 + 2 a N^2 \phi - 2N^3 + a^2 N] \quad .$$

The moment, M_{B-B} , of the actual partition area at the section B - B is:

$$16_p) \quad M_{B-B} = Dp \int_0^{\phi_{\theta}} \int_a^{a+N\delta} r(r \cos(\phi_{\theta} - \delta) - r_{bx}) d\delta dr = dP \int_0^{\phi_{\theta}} [a^2 N \delta + a N^2 \delta^2 + N^3 \delta^3/3] \cos(\phi - \delta) - [a N \delta + N^2 \delta^2/2] r_{bx} d\delta$$

Solving by the method used in 16_o,

$$16_p2) \quad M_{B-B} = dP [N^3 \phi^2 + 2a^2 N \phi - 2N^3 + a^2 N - 2a^2 N \sin \phi] - dP [a N r_{bx} \phi^2/2 + r_{bx} N^2 \phi^3/6 - (2N^3 - a^2 N) \cos \phi] \quad (F_n M_{bb})$$

A partition's moment, M_y , about its plane axis coincident the face surface is:

$$16_q) \quad M_y = \int_0^{\phi_{\theta}} \int_a^{a+N\delta} r^2 \sin(\phi - \delta) d\delta dr = \int_0^{\phi_{\theta}} [a^2 N \delta + a N^2 \delta^2 + N^3 \delta^3/3] \sin(\phi - \delta) d\delta.$$

Again utilizing the math appendix 40 - 43 for solutions of $\int \delta^n \sin(\phi - \delta) d\delta$:

$$\int a^2 N \delta \sin(\phi - \delta) d\delta = a^2 N [\delta \cos(\phi - \delta) + \sin(\phi - \delta)],$$

$$\int a N^2 \delta^2 \sin(\phi - \delta) d\delta = a N^2 [\delta^2 \cos(\phi - \delta) + 2\delta \sin(\phi - \delta) - 2 \cos(\phi - \delta)],$$

$$\int N^3/3 \delta^3 \sin(\phi - \delta) d\delta = N^3/3 [\delta^3 \cos(\phi - \delta) + 3\delta^2 \sin(\phi - \delta) - 6\delta \cos(\phi - \delta) - 6 \sin(\phi - \delta)],$$

and collecting common terms in δ ,

$$[N^3] \delta^2 \sin(\phi - \delta), \quad [2a^2 N] \delta \sin(\phi - \delta), \quad [a^2 N - 2N^3] \sin(\phi - \delta),$$

$$[N^3/3] \delta^3 \cos(\phi - \delta), \quad [a^2 N] \delta^2 \cos(\phi - \delta), \quad [a^2 N - 2N^3] \delta \cos(\phi - \delta), \quad [2a^2 N] \cos(\phi - \delta),$$

equation 16q at the limits $\delta = \phi_{\theta}$ and $\delta = 0$ is:

$$16_r) \quad M_y = dP [N^3 \phi^3/3 + a^2 N \phi^2 + [a^2 N - 2N^3] \phi + [2a^2 N] - [a^2 N - 2N^3] \sin \phi - [2a^2 N] \cos \phi] \quad (F_n M_y)$$

The section A - A moment is:

$$16_s) \quad M_{AY} = [M_{A-A}^2 + M_y^2]^{1/2} \quad (F_n M_{ay})$$

and section B - B moment is:

$$16_u) \quad M_{BY} = [M_{B-B}^2 + M_y^2]^{1/2} \quad (F_n M_{by})$$

The pseudo area moments, M_{A-A}^{pseuso} and M_{B-B}^{pseuso} , and the real area moments M_{AY} and M_{BY} , along with the partition material's T_{max} and σ_{max} are utilized in the next section to find the partition's required thickness at sections A-A and B-B. The composite couples M_{AY} and M_{BY} are utilized with the narrowed section thickness, figure 16 c, in the rectangular beam equations to approximate the stresses at what is, at least at the bearing load radius, a circle arc section.

The ultimate tapered shape of the partition requires a free body stress element, where the forces in the x, y and z axes directions and multitude of couples on either side of the radially extending and the lateral neutral axis are balanced. Given FEA utility and the methods herein, a more accurate analysis is not discussed.

17 THE TAPERED PARTITION.

To find the required partition thickness at sections A-A and B-B, equations 16b and 16i are rearranged. With σ_{max} a partition's maximum tensile stress, the partition thickness required at sections A-A and B-B for the pseudo area are:

17) $t_{A-Apsuedo} = [6 M_{A-Apsuedo} / (x_{A-A} \sigma_{max})]^{1/2}$ (Fn Taap)

17₁) $t_{B-Bpsuedo} = [6 M_{B-Bpsuedo} / (x_{B-B} \sigma_{max})]^{1/2}$ (Fn Tbbp)

Required partition thicknesses for moments M_{ay} and M_{by} are,

17_a) $t_{A-A} = [6 M_{ay} / (x_{A-A} \sigma_{max})]^{1/2}$ (Fn Taa)

17_{a1}) $t_{B-B} = [6 M_{by} / (x_{B-B} \sigma_{max})]^{1/2}$ (Fn Tbb)

Using the pseudo moments and σ_{max} to determine the partitions required thickness at sections A-A and B-B, it's apparent from the solutions in equations 17b and 17c below that angle Γ_1 is greater than Γ_2 and a lower inertia partition with a two step taper is possible.

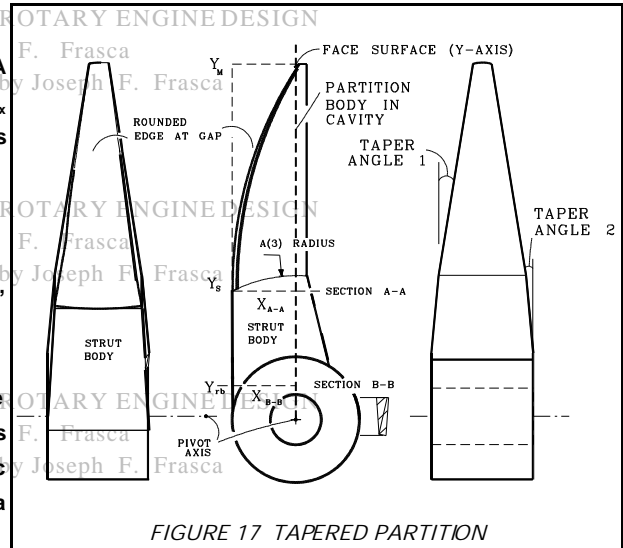


FIGURE 17 TAPERED PARTITION

In the programming, the designer makes a selection from a list of possible partition materials and the program thereafter determines by iteration the maximum loads and moments experienced by a partition during its traverse of the engine cavity's combustion region. Equations 17 and 17a are then used to determine the required partition thickness based on the pseudo area load and actual area load respectively.

In practical engine designs, the partitions have a minimum thickness $b_{(9)}$, at their maximum radius and the second angle starts with initial thickness $b_{1p} = t_{A-Apsuedo}$ for the pseudo approximation and $b_1 = t_{A-A}$ for the close approximation. The partition's angles based on the required thickness by maximum normal stress, σ_{max} , for the pseudo area load are:

17_b) $\Gamma_{1psuedo} = ATN [-b + [(6dP x_s (y_m - y_s)(y_m - y_s)/2) / (x_s \sigma_{max})]^{1/2} / (2y_m - 2y_s)]$ (Fn ptapa)

17_c) $\Gamma_{2psuedo} = ATN [-b_{1p} + [6dP x_s (y_m - y_s) ((y_m + y_s)/2 - r_{bx}) / (x_{B-B} \sigma_{max})]^{1/2} / (2y_s - 2r_{bx})]$.

The partition's angles based on the required thickness by maximum shear stress, τ_{max} , for pseudo area load are:

17_d) $\delta_{1psuedo} = ATN [-b + (3/2 dP [(a + N\phi_{0i}) - a \cos \phi_{0i}] (a \sin \phi_{0i}) / (\tau_{max} x_{A-A})] / (2y_m - 2y_s)]$,

17_e) $\delta_{2psuedo} = ATN [-b_{1p} + (3/2 dP [(a + N\phi_{0i}) - a \cos \phi_{0i}] (a \sin \phi_{0i}) / (\tau_{max} x_{B-B})] / (2y_s - 2r_{bx})]$.

Using the values of M_{ay} and M_{by} from 16t and 16u respectively, the upper and lower angles of partition taper are:

17_f) $\Gamma_1 = ATN [-b + [6M_{ay} / (x_{A-A} \sigma_{max})]^{1/2} / (2y_m - 2y_s)]$, and

17_g) $\Gamma_2 = ATN [-b_1 + [6M_{by} / (x_{B-B} \sigma_{max})]^{1/2} / (2y_s - 2y_{rb})]$.

18 THE PARTITION'S CAVITY VOLUME AND GAP EDGE RELIEF.

Thickness, t, of any point on a partition's tapered portion is: $b_x + 2[r_{max} - r]TAN\epsilon$, where b_x is the partition's minimum thickness, b, and ϵ is the partition taper angle.

For clutter control: $t = b + 2r_{max}TAN\epsilon - 2rTAN\epsilon$, with $A_1 = b + 2r_{max}TAN\epsilon$ and $B_1 = -2TAN\epsilon$; i.e. $t = A_1 + B_1r$.

18) $\int_{in\ cavity} dV = \int_0^{\phi_{0i}} \int_a^{a+N\delta} \int_0^{\phi_{0i}} t r d\theta dr = \int_0^{\phi_{0i}} \int_a^{a+N\delta} [A_1 r + B_1 r^2] d\theta dr = \int_0^{\phi_{0i}} [A_1 r^2/2 + B_1 r^3/3] d\theta = \int_0^{\phi_{0i}} [A_1(aN\delta + N^2\delta^2/2) + B_1(a^2N\delta + aN^2\delta^2 + N^3\delta^3/3)] d\theta$

$\int_0^{\phi_{0i}} dV = \int_0^{\phi_{0i}} [A_1aN + B_1a^2N]\delta + [A_1N^2/2 + B_1aN^2]\delta^2 + [B_1N^3/3]\delta^3 d\theta = [A_1aN + B_1a^2N]\delta^2/2 + [A_1N^2/2 + B_1aN^2]\delta^3/3 + [B_1N^3/3]\delta^4/4 d\theta$.

With constants: $A(106) = [A_1aN + B_1a^2N]/2$, $A(107) = [A_1N^2/6 + B_1aN^2/3]$, and $A(108) = [B_1N^3/12]$, the partition's volume in the annular cavity at rotor angle θ is:

18_a) $PVOL_{0i} = A(106)\phi_{0i}^2 + A(107)\phi_{0i}^3 + A(108)\phi_{0i}^4 + RECA_{(Fn PVOL)}$

PARTITION VOLUME IN RECESS

The constant "RECA" at the end of 18a is the volume of the partition intercept with the face surface recess in those very rare cases when the recess is so shallow that its arc length and depth aren't modified. To find the value of RECA and with reference to figure 18 we have:

$$18_b) \quad RECA = \int_0^{A_2} \int_0^{A(58)} A_1 r + B_1 r^2 d\theta dr - \int_0^{A_1} \int_0^{A(3)} A_1 r + B_1 r^2 d\theta dr + \int_{A_3}^{A_4} \int_0^{d} A_1 r + B_1 r^2 d\theta dr.$$

Using $\int \sec^n \theta d\theta$ solutions from the math appendix 15-19 the solution for RECA $_{(Fn RECA)}$ follows directly as:

$$18_{b2}) \quad RECA = (B_1 d^3 / 6) [SECA_4 TANA_4 + LN^* SECA_4 + TANA_4^*] + (B_1 d^3 / 6) [SECA_3 TANA_3 + LN^* SECA_3 + TANA_3^*] + (A_1 d^2 / 2) TANA_3 + (A_1 A(58)^2 / 2) TANA_2 - (A_1 A(3)^2 / 2) TANA_1 + (B_1 A(58)^3 / 6) [SEC A_2 TAN A_2 + LN^* SEC A_2 + TAN A_2^*] + (B_1 A(3)^3 / 6) [SEC A_1 TAN A_1 + LN^* SEC A_1 + TAN A_1^*].$$

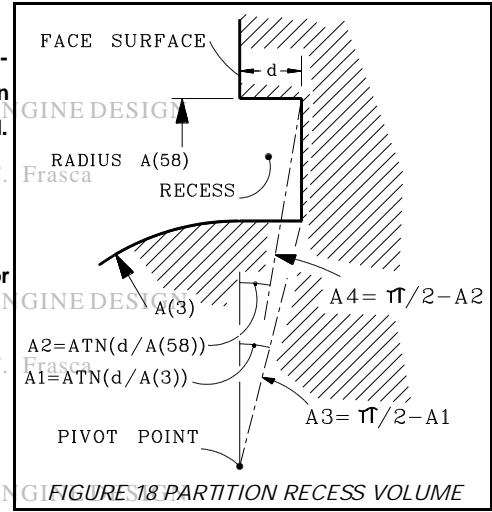


FIGURE 18 PARTITION RECESS VOLUME

PARTITION GAP EDGE RELIEF

For a reasonable approximation of the material removed at the partition gap edge to clear the wave surface, the camber angle ϵ of the edge is set equal to arctangent of dy/dx found in equation 13e; i.e. $\epsilon = ATN(dy/dx)$. With reference to figure 18a, the volume increment removed from the partition edge is $dV = 2 \left(\frac{t}{2} TAN \epsilon \right) \left[\frac{t}{2} \right] / 2$, and thickness t in dV is $[A_1 + B_1 r]$. The partition's wave surface clearance volume, REL, is:

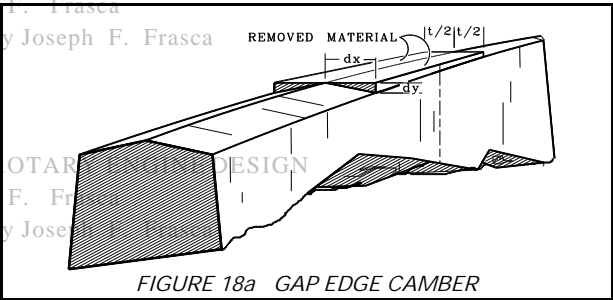


FIGURE 18a GAP EDGE CAMBER

$$18_c) \quad REL = \int (t/2)^2 TAN^2 \epsilon dS = 1/4 \int [A_1^2 + 2A_1 B_1 r + B_1^2 r^2] TAN^2 \epsilon dS = 1/4 \int [A_1^2 + 2A_1 B_1 (a + N\delta) + B_1^2 (a^2 + 2aN\delta + N^2 \delta^2)] TAN^2 \epsilon dS.$$

For clutter control: $C_1 = 1/4 TAN^2 \epsilon [A_1^2 + 2A_1 B_1 a + B_1^2 a^2]$, $C_2 \delta = 1/4 TAN^2 \epsilon [2A_1 B_1 N + 2aB_1^2] \delta$, and $C_3 \delta^2 = 1/4 TAN^2 \epsilon [N^2 B_1^2] \delta^2$.

Using $r = a + N\delta$, $dr = N d\delta$ and $dS = [dr^2 + (rd\delta)^2]^{1/2}$,

$$18_d) \quad REL = \int [C_1 + C_2 \delta + C_3 \delta^2] [N^2 d\delta^2 + r^2 d\delta^2]^{1/2}.$$

Using $NTAN\mu = r = a + N\delta$, $\delta = (NTAN\mu - a)/N$ and $d\delta = SEC^2 \mu d\mu$.

$$18_e) \quad REL = \int [C_1 + C_2 \delta_{\theta_1} + C_3 \delta_{\theta_1}^2] [N^2 d\delta^2 + N^2 TAN^2 \mu d\delta^2]^{1/2} = \int [C_1 + C_2 \delta_{\theta_1} + C_3 \delta_{\theta_1}^2] N SEC^3 \mu d\mu$$

$$18_f) \quad REL = \int [C_1 + C_2 (TAN\mu - a/N) + C_3 (TAN^2 \mu - 2aTAN\mu/N + a^2/N^2)] N SEC^3 \mu d\mu.$$

For clutter control: $C_a = NC_1 - aC_2 + C_3 a^2/N$, $C_b TAN\mu = [NC_2 - 2C_3 a] TAN\mu$, $C_c TAN^2 \mu = NC_3 TAN^2 \mu$.

$$18_g) \quad REL = \int [C_a + C_b TAN\mu + C_c TAN^2 \mu] SEC^3 \mu d\mu$$

Using the math appendix 15 - 19 for $\int \sec^n \theta d\theta$ solutions:

$$\int C_a SEC^3 \mu d\mu = C_a/2 [SEC\mu TAN\mu + LN^* SEC\mu + TAN\mu^*], \quad \int C_b TAN\mu SEC^3 \mu d\mu = C_b/3 SEC^3 \mu, \quad \int C_c TAN^2 \mu SEC^3 \mu d\mu = \int C_c [SEC^5 \mu - SEC^3 \mu] d\mu = C_c/4 SEC^3 \mu TAN\mu - C_c/8 [SEC\mu TAN\mu + LN^* SEC\mu + TAN\mu^*].$$

Collecting common coefficients of terms in μ :

$$[C_a/2 - C_c/8] SEC\mu TAN\mu = A(115) TAN\mu SEC\mu, \quad [C_b/2 - C_c/8] LN^* SEC\mu + TAN\mu^* = A(116) LN^* SEC\mu + TAN\mu^* \\ [C_a/3] SEC^3 \mu = A(117) SEC^3 \mu, \quad [C_c/4] SEC^3 \mu TAN\mu = A(118) SEC^3 \mu TAN\mu.$$

In 18g, then, with $\delta = 0$ the lower limit $\mu = ATN(a/N)$ and with $\delta = \phi_{\theta_1}$ the upper limit is $\mu = ATN(a/N + \phi_{\theta_1})$.

$$18_h) \quad REL = \frac{A(115) TAN\mu SEC\mu}{ATN(a/N)} + \frac{A(116) LN^* SEC\mu + TAN\mu^*}{ATN(a/N)} + \frac{A(117) SEC^3 \mu}{ATN(a/N)} + \frac{A(118) SEC^3 \mu TAN\mu}{ATN(a/N)}.$$

The constant A(119) is equated to the negative value of 18h above evaluated at $\mu = ATN(a/N)$ and

$$18_i) \quad REL = \frac{A(115) TAN\mu SEC\mu}{ATN(a/N + \delta_{\theta_1})} + \frac{A(116) LN^* SEC\mu + TAN\mu^*}{ATN(a/N + \delta_{\theta_1})} + \frac{A(117) SEC^3 \mu}{ATN(a/N + \delta_{\theta_1})} + \frac{A(118) SEC^3 \mu TAN\mu}{ATN(a/N + \delta_{\theta_1})} + A(119).$$

A reasonably accurate partition volume in the annular cavity at θ_i is now:

$$18_j) \quad PVOL_{\theta_i} = A(106) \phi_{\theta_i}^2 + A(107) \phi_{\theta_i}^3 + A(108) \phi_{\theta_i}^4 + RECA - REL_{(Fn PVOL)}.$$