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APPENDIX A : Math Appendix

$\int \mathrm{COS}^n\theta\, d\theta$

1 $\int \mathrm{COS}\,\theta\, d\theta = \mathrm{SIN}\,\theta.$

2 $\int \mathrm{COS}^2\theta\, d\theta = \int 1/2\,(1 + \mathrm{COS}\,2\theta)\,d\theta = 1/2\,\theta + 1/4\,\mathrm{SIN}\,2\theta.$

3 $\int \mathrm{COS}^3\theta\, d\theta = \int \mathrm{COS}\,\theta\,(1 - \mathrm{SIN}^2\theta)\,d\theta = \mathrm{SIN}\,\theta - 1/3\,\mathrm{SIN}^3\theta.$

4 $\int \mathrm{COS}^4\theta\, d\theta = \int 1/4\,[1 + \mathrm{COS}2\theta]^2 d\theta = \int 1/4\; 1 + 2\,\mathrm{COS}\,2\theta + \mathrm{COS}^2 2\theta\,]\,d\theta$

$= \theta/4 + 1/4\,\mathrm{SIN}2\theta + 1/4\,[\int \mathrm{COS}^2 2\theta\,d\theta = 1/2\int(1 + \mathrm{COS}4\theta)\,d\theta = \theta/2 + 1/8\,\mathrm{SIN}4\theta\,]$

$= 3/8\,\theta + 1/4\,\mathrm{SIN}\,2\theta + 1/32\,\mathrm{SIN}\,4\theta.$

5 $\int \mathrm{COS}^5\theta\, d\theta = \int \mathrm{COS}\,\theta\,[1 - 2\,\mathrm{SIN}^2\theta + \mathrm{SIN}^4\theta\,]\,d\theta = \mathrm{SIN}\,\theta - 2/3\,\mathrm{SIN}3\theta + 1/5\,\mathrm{SIN}5\theta.$

6 $\int \mathrm{COS}^6\theta\, d\theta = \int 1/8\,[1 + \mathrm{COS}2\theta\,]^3\,d\theta = \int 1/8\,[1 + 3\,\mathrm{COS}2\theta + 3\,\mathrm{COS}^2 2\theta + \mathrm{COS}^3 2\theta\,]\,d\theta$

$= 1/8\theta + 3/16\,\mathrm{SIN}2\theta + \int 3/16\int[1 + \mathrm{COS}4\theta\,]\,d\theta + 1/8\int \mathrm{COS}2\theta\,[1 - \mathrm{SIN}^2 2\theta\,]\,d\theta\,]$

$= 1/8\theta + 3/16\,\mathrm{SIN}2\theta + [\,3/16\theta + 3/64\,\mathrm{SIN}4\theta + 1/16\,\mathrm{SIN}2\theta - 1/48\,\mathrm{SIN}^3 2\theta\,]$

$= 5/16\theta + 1/4\,\mathrm{SIN}2\theta + 3/64\,\mathrm{SIN}4\theta - 1/48\,\mathrm{SIN}^3 2\theta.$

7 $\int \mathrm{COS}^7\theta\, d\theta = \int \mathrm{COS}\theta\,[1 - \mathrm{SIN}^2\theta\,]^3\,d\theta = \int \mathrm{COS}\theta\,[1 - 3\,\mathrm{SIN}^2\theta + 3\,\mathrm{SIN}^4\theta + \mathrm{SIN}^6\theta\,]\,d\theta$

$= \mathrm{SIN}\theta - \mathrm{SIN}^3\theta + 3/5\,\mathrm{SIN}^5\theta + 1/7\,\mathrm{SIN}^7\theta$

$\int \mathrm{SIN}^n\, d\theta$

8 $\int \mathrm{SIN}\theta\, d\theta = -\,\mathrm{COS}\theta$

9 $\int \mathrm{SIN}^2\theta\, d\theta = \int 1/2\,(1 - \mathrm{COS}\,2\theta\,)\,d\theta = 1/2\,\theta - 1/4\,\mathrm{SIN}2\theta.$

10 $\int \mathrm{SIN}^3\theta\, d\theta = \int \mathrm{SIN}\,\theta\,(1 - \mathrm{COS}^2\theta\,)\,d\theta = -\,\mathrm{COS}\theta + 1/3\,\mathrm{COS}^3\theta.$

11 $\int \mathrm{SIN}^4\theta\, d\theta = \int 1/4\,[\,1 - \mathrm{COS}2\theta\,]^2\,d\theta = \int 1/4\,[\,1 - 2\,\mathrm{COS}2\theta + \mathrm{COS}^2 2\theta\,]\,d\theta$

$= \theta/4 - 1/4\,\mathrm{SIN}2\theta + 1/4\,[\int \mathrm{COS}^2 2\theta\,d\theta = 1/2\int(1 + \mathrm{COS}\,4\theta\,)\,d\theta = \theta/2 + 1/8\,\mathrm{SIN}4\theta\,]$

$= 3/8\,\theta - 1/4\,\mathrm{SIN}\,2\theta + 1/32\,\mathrm{SIN}\,4\theta.$

12 $\int \mathrm{SIN}^5\theta\, d\theta = \int \mathrm{SIN}\,\theta\,[\,1 - 2\,\mathrm{COS}^2\theta + \mathrm{COS}^4\theta\,]\,d\theta = -\,\mathrm{COS}\theta + 2/3\,\mathrm{COS}^3\theta - 1/5\,\mathrm{SIN}^5\theta.$

13 $\int \mathrm{SIN}^6\theta\, d\theta = \int 1/8\,[\,1 - \mathrm{COS}\,2\theta\,]^3\,d\theta = \int 1/8\,[\,1 - 3\,\mathrm{COS}2\theta + 3\,\mathrm{COS}^2 2\theta - \mathrm{COS}^3 2\theta\,]\,d\theta$

$= 1/8\theta - 3/16\,\mathrm{SIN}2\theta + \int 3/16\int[\,1 + \mathrm{COS}4\theta\,]\,d\theta - 1/8\int \mathrm{COS}\,2\theta\,[\,1 - \mathrm{SIN}^2 2\theta\,]\,d\theta\,]$

$= 1/8\theta + 3/16\,\mathrm{SIN}2\theta + [\,3/16\theta + 3/64\,\mathrm{SIN}4\theta - 1/16\,\mathrm{SIN}2\theta + 1/48\,\mathrm{SIN}^3 2\theta\,]$

$= 5/16\theta + 1/8\,\mathrm{SIN}2\theta + 3/64\,\mathrm{SIN}4\theta + 1/48\,\mathrm{SIN}^3 2\theta$

14 $\int \mathrm{SIN}^7\theta\, d\theta = \int \mathrm{SIN}\theta\,[\,1 - \mathrm{COS}^2\theta\,]^3\,d\theta = \int \mathrm{SIN}\theta\,[\,1 - 3\,\mathrm{COS}^2\theta + 3\,\mathrm{COS}^4\theta - \mathrm{COS}^6\theta\,]\,d\theta$

$= -\,\mathrm{COS}\theta + \mathrm{COS}^3\theta - 3/5\,\mathrm{COS}^5\theta + 1/7\,\mathrm{COS}^7\theta.$

$\int \mathrm{SEC}^n\theta\, d\theta$

15 $\int \mathrm{SEC}\,\theta\, d\theta = \int \mathrm{SEC}\theta\cdot[\,\mathrm{SEC}\theta + \mathrm{TAN}\theta\,]/[\,\mathrm{SEC}\theta + \mathrm{TAN}\theta\,]\,d\theta = \mathrm{Ln}{*}\,\mathrm{SEC}\theta + \mathrm{TAN}\theta{*}.$

16 $\int \mathrm{SEC}^2\theta\, d\theta = \mathrm{TAN}\theta.$

17 $\int \mathrm{SEC}^3\theta\, d\theta = \mathrm{SEC}\theta\,\mathrm{TAN}\theta - [\int \mathrm{SEC}\theta\,\mathrm{TAN}^2\theta\,d\theta = \int(\,\mathrm{SEC}^3\theta - \mathrm{SEC}\theta\,)\,d\theta\,] = 1/2\,[\,\mathrm{SEC}\theta\,\mathrm{TAN}\theta + \ln{*}\mathrm{SEC}\theta + \mathrm{TAN}\theta{*}\,].$

18 $\int \mathrm{SEC}^4\theta\, d\theta = \int \mathrm{SEC}^2\theta\cdot[\,1 + \mathrm{TAN}^2\theta\,]\,d\theta = \mathrm{TAN}\theta + 1/3\,\mathrm{TAN}^3\theta.$

19 $\int \mathrm{SEC}^5\theta\, d\theta = \mathrm{SEC}^3\theta\,\mathrm{TAN}\theta - 3\,[\int \mathrm{SEC}^3\theta\,\mathrm{TAN}^2\theta\,d\theta = \int \mathrm{SEC}^5\theta\,d\theta - \int \mathrm{SEC}^3\theta\,d\theta\,]$

$= 1/4\,\mathrm{SEC}^3\theta\,\mathrm{TAN}\theta + 3/8\,[\,\mathrm{SEC}\theta\,\mathrm{TAN}\theta + \ln{*}\mathrm{SEC}\theta + \mathrm{TAN}\theta{*}\,].$

$\int \mathrm{TAN}^n\theta\, d\theta$

20 $\int \mathrm{TAN}\,\theta\, d\theta = -\,\mathrm{Ln}{*}\,\mathrm{COS}\theta{*}.$

21 $\int \mathrm{TAN}^2\theta\, d\theta = \int \mathrm{SEC}^2\theta - 1\,]\,d\theta = \mathrm{TAN}\,\theta - \theta.$

22 $\int \mathrm{TAN}^3\theta\, d\theta = \int \mathrm{TAN}\theta\,[\,\mathrm{SEC}^2\theta - 1\,]\,d\theta = 1/2\,\mathrm{TAN}^2\theta + \mathrm{Ln}{*}\,\mathrm{COS}\theta{*}.$

23 $\int \mathrm{TAN}^4\theta\, d\theta = \int \mathrm{TAN}^2\theta\,(\,\mathrm{SEC}^2\theta - 1\,)\,d\theta = 1/3\,\mathrm{TAN}^3\theta - \mathrm{TAN}\,\theta + \theta.$

24 $\int \mathrm{TAN}^5\theta\, d\theta = \int \mathrm{TAN}^3\theta\,[\,\mathrm{SEC}^2\theta - 1\,]\,d\theta = 1/4\,\mathrm{TAN}^4\theta - 1/2\,\mathrm{TAN}^2\theta - \mathrm{Ln}{*}\,\mathrm{COS}\theta{*}.$

25 $\int \mathrm{TAN}^6\theta\, d\theta = \int \mathrm{TAN}^4\theta\,[\,\mathrm{SEC}^2\theta - 1\,]\,d\theta = 1/5\,\mathrm{TAN}^5\theta - 1/3\,\mathrm{TAN}^3\theta + \mathrm{TAN}\,\theta - \theta.$

26 $\int \mathrm{TAN}^7\theta\, d\theta = \int \mathrm{TAN}^5\theta\,[\,\mathrm{SEC}^2\theta - 1\,]\,d\theta = 1/6\,\mathrm{TAN}^6\theta - 1/4\,\mathrm{TAN}^4\theta + 1/2\,\mathrm{TAN}^2\theta + \mathrm{Ln}{*}\mathrm{COS}\theta{*}$

$\int \mathrm{SIN}^2\theta\,\mathrm{TAN}^n\theta\, d\theta$

27 $\int \mathrm{SIN}^2\theta\,\mathrm{TAN}\theta\, d\theta = \int \mathrm{TAN}\theta - \mathrm{TAN}\theta\,\mathrm{COS}^2\theta\,d\theta = -\,\mathrm{Ln}{*}\,\mathrm{COS}\theta{*} + 1/2\,\mathrm{COS}^2\theta.$

28 $\int \mathrm{SIN}^2\theta\,\mathrm{TAN}^2\theta\, d\theta = \int \mathrm{TAN}^2\theta - \mathrm{TAN}^2\theta\,\mathrm{COS}^2\theta\,d\theta = \mathrm{TAN}\theta - \theta - 1/2\,(\,\theta - 1/2\,\mathrm{SIN}2\theta\,) = -\,3/2\,\theta + \mathrm{TAN}\theta + 1/4\,\mathrm{SIN}2\theta.$

29 $\int \mathrm{SIN}^2\theta\,\mathrm{TAN}^3\theta\, d\theta = \int \mathrm{TAN}^3\theta - \mathrm{TAN}^3\theta\,\mathrm{COS}^2\theta\,d\theta = 1/2\,\mathrm{TAN}^2\theta + 2\mathrm{Ln}{*}\mathrm{COS}\theta{*} - 1/2\,\mathrm{COS}^2\theta.$

$\int \mathrm{TAN}^n\theta\,\mathrm{SIN}\theta\,\mathrm{COS}\theta\, d\theta$

30 $\int \mathrm{TAN}^2\theta\,\mathrm{SIN}\theta\mathrm{OS}\theta\, d\theta = \int \mathrm{TAN}\theta\,\mathrm{SIN}^2\theta\,d\theta = \int \mathrm{TAN}\theta\,[\,1 - \mathrm{COS}^2\theta\,]\,d\theta = -\,\mathrm{Ln}{*}\,\mathrm{COS}\theta{*} + 1/2\,\mathrm{COS}^2\theta.$

31 $\int \text{TAN}^3\theta\ \text{SIN}\theta\ \text{COS}\theta\ d\theta = \int \text{TAN}^2\theta\ \text{SIN}^2\theta\ d\theta = \int \text{TAN}^2\theta\ [1 - \text{COS}^2\theta]\ d\theta = -3/2\theta + 1/4\ \text{SIN}2\theta + \text{TAN}\ \theta.$

32 $\int \text{TAN}^4\theta\ \text{SIN}\theta\ \text{COS}\theta\ d\theta = \int \text{TAN}^3\theta\ \text{SIN}^2\theta\ d\theta = \int \text{TAN}^3\theta\ [1 - \text{COS}^2\theta]\ d\theta = \int \text{TAN}^3\theta\ d\theta - \int \text{TAN}\theta\ \text{SIN}^2\theta\ d\theta$

$= 1/2\ \text{TAN}^2\theta + 2\text{Ln}^*\ \text{COS}\theta^* - 1/2\ \text{COS}^2\theta.$

33 $\int \text{TAN}^5\theta\ \text{SIN}\theta\ \text{COS}\theta\ d\theta = \int \text{TAN}^4\theta\ \text{SIN}^2\theta\ d\theta = \int \text{TAN}^4\theta\ [1 - \text{COS}^2\theta]\ d\theta = \int \text{TAN}^4\theta\ d\theta - \int \text{TAN}^2\theta\ d\theta + \int \text{SIN}^2\theta\ d\theta$

$= 1/3\ \text{TAN}^3\theta - 2\ \text{TAN}\theta + 5/2\ \theta - 1/4\ \text{SIN}2\theta.$

34 $\int \text{TAN}^6\theta\ \text{SIN}\theta\ \text{COS}\theta\ d\theta = \int \text{TAN}^5\theta\ \text{SIN}^2\theta\ d\theta = \int \text{TAN}^5\theta\ d\theta - \int \text{TAN}^3\theta\ d\theta + \int \text{TAN}\theta\ d\theta - \int \text{SIN}\theta\ \text{COS}\theta\ d\theta$

$= 1/4\ \text{TAN}^4\theta - \text{TAN}^2\theta - 3\text{Ln}^*\ \text{COS}\theta^* + 1/2\ \text{COS}^2\theta.$

$\int \theta\ \text{TAN}^n\theta\ \text{SEC}^2\theta\ d\theta$

35 $\int \theta\ \text{TAN}^2\theta\ d\theta = \int \theta\ (\text{SEC}^2\theta - 1)\ d\theta = -\theta^2/2 + \theta\ \text{TAN}\theta + \text{Ln}^*\text{COS}\theta^*.$

36 $\int \theta\ \text{TAN}\theta\ \text{SEC}^2\theta\ d\theta = \theta/2\ \text{TAN}^2\theta - 1/2\int \text{TAN}^2\theta\ d\theta = \theta/2\ \text{TAN}^2\theta - 1/2\ \text{TAN}\theta + 1/2\ \theta.$

37 $\int \theta\text{TAN}^2\theta\ \text{SEC}^2\theta\ d\theta = \theta\ \text{TAN}^3\theta/3 - 1/3\ [\int \text{TAN}^3\theta\ d\theta = 1/2\ \text{TAN}^2\theta + \text{Ln}^*\text{COS}\theta^*]$

$= \theta/3\ \text{TAN}^3\theta - 1/6\ \text{TAN}^2\theta - 1/3\text{Ln}^*\ \text{COS}\theta^*.$

38 $\int \theta\text{TAN}^3\theta\ \text{SEC}^2\theta\ d\theta = \theta/4\ \text{TAN}^4\theta - 1/4\ [\int \text{TAN}^4\theta\ d\theta = 1/3\ \text{TAN}^3\theta - \text{TAN}\theta + \theta]$

$= \theta/4\ \text{TAN}^4\theta - 1/12\ \text{TAN}^3\theta + 1/4\ \text{TAN}\theta - 1/4\theta.$

39 $\int \theta\ \text{TAN}^4\theta\ \text{SEC}^2\theta\ d\theta = \theta\ \text{TAN}^5\theta/5 - 1/5\ [\int \text{TAN}^5\theta\ d\theta = 1/4\ \text{TAN}^4\theta - 1/2\ \text{TAN}^2\theta - \text{Ln}^*\text{COS}\theta^*]$

$= \theta/5\ \text{TAN}^5\theta - 1/20\ \text{TAN}^4\theta + 1/10\ \text{TAN}^2\theta + 1/5\ \text{Ln}^*\text{COS}\theta^*.$

$\int \sigma^n\ \text{SIN}\ (B - \sigma)\ d\sigma$

40 $\int \sigma\ \text{SIN}\ (B - \sigma)\ d\sigma = \underline{\sigma\ \text{COS}\ (B - \sigma)} + [-\int \text{COS}\ (B - \sigma)\ d\sigma = \underline{\text{SIN}\ (B - \sigma)}] = \underline{\sigma\ \text{COS}\ (B - \sigma) + \text{SIN}\ (B - \sigma)}.$

41 $\int \sigma^2\ \text{SIN}\ (B - \sigma)\ d\sigma = \underline{\sigma^2\ \text{COS}\ (B - \sigma)} + [-\int 2\sigma\ \text{COS}\ (B - \sigma)d\sigma = \underline{2\sigma\ \text{SIN}\ (B - \sigma)} + [\int -2\ \text{SIN}\ (B - \sigma)d\sigma = \underline{-2\ \text{COS}\ (B - \sigma)}]]$

$= \sigma^2\ \text{COS}\ (B - \sigma) + 2\sigma\ \text{SIN}\ (B - \sigma) - 2\ \text{COS}\ (B - \sigma).$

42 $\int \sigma^3\ \text{SIN}\ (B - \sigma)d\sigma$

$= \underline{\sigma^3\ \text{COS}\ (B - \sigma)} + [-\int 3\sigma^2\ \text{COS}\ (B - \sigma)d\sigma = \underline{3\sigma^2\ \text{SIN}\ (B - \sigma)} + [\int -6\sigma\ \text{SIN}\ (B - \sigma)d\sigma = \underline{-6\sigma\ \text{COS}\ (B - \sigma) - 6\ \text{SIN}\ (B - \sigma)}]]$

$= \sigma^3\ \text{COS}\ (B - \sigma) + 3\sigma^2\ \text{SIN}\ (B - \sigma) - 6\sigma\ \text{COS}\ (B - \sigma) - 6\ \text{SIN}\ (B - \sigma).$

43 $\int \sigma^4\text{SIN}(B - \sigma)d\sigma = \underline{\sigma^4\text{COS}(B-\sigma)} + [-\int 4\sigma^3\text{COS}(B-\sigma)d\sigma = \underline{4\sigma^3\text{SIN}(B-\sigma)} + [-\int 12\sigma^2\ \text{SIN}(B-\sigma)d\sigma = \underline{-12\sigma^2\text{COS}(B-\sigma) - 24\sigma\text{SIN}(B-\sigma) + 24\ \text{COS}(B-\sigma)}$

$= \sigma^4\ \text{COS}\ (B - \sigma) + 4\sigma^3\ \text{SIN}(B - \sigma) - 12\sigma^2\text{COS}(B - \sigma) - 24\sigma\ \text{SIN}(B - \sigma) + 24\ \text{COS}(B - \sigma).$

$\int \sigma^n\ \text{COS}\ (B - \sigma)\ d\sigma$

44 $\int \sigma\ \text{COS}(B - \sigma)\ d\sigma = -\sigma\ \text{SIN}\ (B - \sigma) + [\int \text{SIN}(B - \sigma)\ d\sigma = \text{COS}\ (B - \sigma)] = -\sigma\ \text{SIN}\ (B - \sigma) + \text{COS}\ (B - \sigma).$

45 $\int \sigma^2\text{COS}(B - \sigma)\ d\sigma = -\sigma^2\ \text{SIN}\ (B - \sigma) + \{\int 2\sigma\ \text{SIN}\ (B - \sigma)\ d\sigma = 2\sigma\ \text{COS}\ (B - \sigma) + [-\int 2\ \text{COS}\ (B - \sigma)\ d\sigma = 2\ \text{SIN}\ (B - \sigma)]\}$

$= -\sigma^2\ \text{SIN}\ (B - \sigma) + 2\sigma\ \text{COS}\ (B - \sigma) + 2\ \text{SIN}\ (B - \sigma).$

46 $\int \sigma^3\text{COS}(B - \sigma)\ d\sigma = -\sigma^3\ \text{SIN}\ (B - \sigma) +$

$\{\int 3\sigma 2\ \text{SIN}\ (B - \sigma)d\sigma = 3\sigma^2\ \text{COS}\ (B - \sigma) + [-\int 6\sigma\ \text{COS}\ (B - \sigma)d\sigma = 6\sigma\ \text{SIN}\ (B - \sigma) + [-\int 6\ \text{SIN}\ (B - \sigma)d\sigma = -6\ \text{COS}\ (B - \sigma)]]\}$

$= -\sigma^3\ \text{SIN}\ (B - \sigma) + 3\sigma^2\ \text{COS}\ (B - \sigma) + 6\sigma\ \text{SIN}\ (B - \sigma) - 6\ \text{COS}\ (B - \sigma).$

47 $\int \sigma^4\text{COS}(B - \sigma)\ d\sigma = \underline{-\sigma^4\ \text{SIN}\ (B - \sigma)} +$

$\{4\int \sigma^3\ \text{COS}\ (B - \sigma)\ d\sigma = 4\sigma^3\ \text{COS}\ (B - \sigma) + 12\sigma^2\ \text{SIN}\ (B - \sigma) - 24\sigma\ \text{COS}\ (B - \sigma) - 24\ \text{SIN}\ (B - \sigma)\}$

$= -\sigma^4\ \text{SIN}(B - \sigma) + 4\sigma^3\text{COS}(B - \sigma) + 12\sigma^2\text{SIN}(B - \sigma) - 24\sigma\text{COS}(B - \sigma) - 24\ \text{SIN}(B - \sigma).$

$\int \sigma^n\ \text{COS}2(B - \sigma)\ d\sigma$

48 $\int \sigma\ \text{COS}2(B - \sigma)\ d\sigma = -\sigma/2\ \text{SIN}\ 2\ (B - \sigma) + \{\int 1/2\ \text{SIN}\ 2\ (B - \sigma)\ d\sigma = 1/4\ \text{COS}\ 2\ (B - \sigma)\}$

$= -\sigma/2\ \text{SIN}2\ (B - \sigma) + 1/4\ \text{COS}2\ (B - \sigma).$

49 $\int \sigma^2\text{COS}2(B - \sigma)\ d\sigma =$

$\underline{-\sigma^2/2\ \text{SIN}\ 2\ (B - \sigma)} + \{\int \sigma\ \text{SIN}\ 2\ (B - \sigma)\ d\sigma = \underline{\sigma/2\ \text{COS}\ 2\ (B - \sigma)} + [-\int 1/2\ \text{COS}\ 2\ (B - \sigma)\ d\sigma = \underline{1/4\ \text{SIN}\ 2\ (B - \sigma)}]\}$

$= -\sigma^2/2\ \text{SIN}2\ (B - \sigma) + \sigma/2\ \text{COS}2\ (B - \sigma) + 1/4\ \text{SIN}2\ (B - \sigma).$

50 $\int \sigma^3\text{COS}2(B - \sigma)\ d\sigma$

$= \underline{-\sigma^3/2\ \text{SIN}\ 2\ (B - \sigma)} + \{\int 3/2\ \sigma^2\ \text{SIN}\ 2\ (B - \sigma)d\sigma = \underline{3\sigma^2/4\ \text{COS}\ 2\ (B - \sigma)} + {}^{A}[-3/2\int \sigma\ \text{COS}\ 2\ (B - \sigma)\ d\sigma].$

$A\ \{-3/2\int \sigma\ \text{COS}\ 2\ (B - \sigma)\ d\sigma = + \underline{3\sigma/4\ \text{SIN}2\ (B - \sigma) - 3/8\ \text{COS}2\ (B - \sigma)}]$

$= -\sigma^3/2\ \text{SIN}\ 2\ (B - \sigma) + 3/4\ \sigma^2\text{COS}\ 2(B - \sigma) + 3/4\ \sigma\text{SIN}\ 2(B - \sigma) - 3/8\ \text{COS}\ 2(B - \sigma).$

51 $\int \sigma^4\text{COS}2(B - \sigma)\ d\sigma = \underline{-\sigma^4/2\ \text{SIN}\ 2\ (B - \sigma)} + \{\int 2\sigma^3\ \text{SIN}\ 2\ (B - \sigma)\ d\sigma = \underline{\sigma^3\ \text{COS}\ 2\ (B - \sigma)} - {}^{A}\int 3\sigma^2\ \text{COS}\ 2\ (B - \sigma)\ d\sigma\}$

$A\ \{\int -3\sigma^2\ \text{COS}\ 2\ (B - \sigma)\ d\sigma = \underline{3\sigma^2/2\ \text{SIN}\ 2\ (B - \sigma) - 3\sigma/2\ \text{COS}\ 2\ (B - \sigma) - 3/4\ \text{SIN}\ 2\ (B - \sigma)}]$

$= -\sigma^4/2\ \text{SIN}2\ (B - \sigma) + \sigma^3\text{COS}2\ (B - \sigma) + 3\sigma^2/2\ \text{SIN}2(B - \sigma) - 3\sigma/2\ \text{COS}2(B - \sigma) - 3/4\ \text{SIN}2(B - \sigma).$

$\int \theta^n\ \text{SIN}\ \theta\ d\theta$

52 $\int \theta\ \text{SIN}\theta\ d\theta = -\theta\ \text{COS}\ (\theta) + \int \text{COS}\ (\theta)\ d\theta = -\theta\ \text{COS}\ \theta + \text{SIN}\ \theta.$

53 $\int \theta^2\ \text{SIN}\theta\ d\theta = -\theta^2\ \text{COS}\ (\theta) + [2\int \theta\ \text{COS}\ (\theta)\ d\theta = 2\ [\theta\ \text{SIN}\ (\theta) + \text{COS}\ (\theta)]] = -\theta^2\ \text{COS}\ \theta + 2\theta\text{SIN}\theta + 2\ \text{COS}\ \theta.$

54 $\int \theta^3\ \text{SIN}\theta\ d\theta = -\theta^3\ \text{COS}\ (\theta) + [3\int \theta^2\ \text{COS}\ (\theta)\ d\theta = 3\theta^2\ \text{SIN}\ (\theta) + 6\theta\ \text{COS}\ (\theta) - 6\ \text{SIN}\ (\theta)]$

$= -\theta^3\ \text{COS}\theta + 3\theta^2\ \text{SIN}\theta + 6\theta\ \text{COS}\theta - 6\ \text{SIN}\theta.$

55 $\int \theta^4\ \text{SIN}\theta\ d\theta = -\theta^4\ \text{COS}\theta + 4\ [\int \theta^3\ \text{COS}\ (\theta)\ d\theta = \theta^3\ \text{SIN}\ (\theta) + 3\theta^2\cdot\ \text{COS}\ (\theta) - 6\theta\cdot\ \text{SIN}\ (\theta) - 6\ \text{COS}\ (\theta)]$

$= -\theta^4\ \text{COS}\theta + 4\theta^3\ \text{SIN}\theta + 12\theta^2\ \text{COS}\theta - 24\theta\ \text{SIN}\theta - 24\ \text{COS}\theta.$

$\int \theta^n \cos\theta \, d\theta$

56 $\int \theta \cos\theta \, d\theta = \theta \sin\theta - \int \sin\theta \, d\theta = \theta \sin(\theta) + \cos\theta.$

57 $\int \theta^2 \cos\theta \, d\theta = \theta^2 \sin\theta - [2\int \theta \sin\theta \, d\theta = 2[-\theta\cos\theta + \sin\theta]] = \theta^2 \sin\theta + 2\cdot\theta\cos\theta - 2\sin\theta.$

58 $\int \theta^3 \cos\theta \, d\theta = \theta^3 \sin(\theta) - [3\int \theta^2 \sin(\theta)\, d\theta = 3(-\theta^2\cos(\theta) + 2\theta\sin(\theta) + 2\cos(\theta))]$

$= \theta^3 \sin\theta + 3\theta^2\cos\theta - 6\theta\sin\theta - 6\cos\theta.$

59 $\int \theta^4 \cos\theta \, d\theta = \theta^4 \sin(\theta) - 4[\int \theta^3 \sin(\theta)\, d\theta = \theta^3\cos(\theta) + 3\theta^2 \sin(\theta) + 6\theta\cos(\theta) - 6\cdot\sin(\theta)]$

$= \theta^4 \sin\theta + 4\cdot\theta^3\cos\theta - 12\cdot\theta^2\sin\theta - 24\,\theta\cos\theta + 24\sin\theta.$

$\int \cos^n u\theta \sin\theta \, d\theta$

$r_0 = 1/u$, $r_1 = 1/u^2$, $r_2 = 1/(u^2 - 1)$, $r_3 = 1/(1 - 9u^2)$, $r_4 = 1/(1 - 16u^2)$, $r_5 = 1/(1 - 25u^2)$,
$r_6 = 1/(1 - 36u^2)$, $r_7 = 1/(1 - 49u^2)$, $s_0 = 1/(2u)$, $s_1 = 1/(4u^2)$, and $s_2 = 1/(4u^2 - 1)$.

60 $\int \cos u\theta \sin\theta \, d\theta = \underline{1/u \sin u\theta \sin\theta} + [-1/u\int \sin u\theta \cos\theta \, d\theta = \underline{1/u^2 \cos u\theta \cos\theta} - 1/u^2\int \cos u\theta(-\sin\theta)\, d\theta]$

$= u^2/(u^2 - 1)[1/u \sin u\theta \sin\theta + 1/u^2 \cos u\theta \cos\theta] = r_2[u \sin u\theta \sin\theta + \cos u\theta \cos\theta].$

61 $\int \cos^2 u\theta \sin\theta \, d\theta = \int[1/2 + 1/2\cos 2u\theta]\sin\theta\, d\theta = \underline{-1/2\cos\theta} + 1/2\,{}^A[\int \cos 2u\theta \sin\theta \, d\theta]$

$A\{\int \cos 2u\theta \sin\theta\, d\theta = \underline{1/(2u)\sin 2u\theta \sin\theta} + [-\int 1/(2u)\sin 2u\theta\cos\theta\, d\theta = \underline{1/(4u^2)\cos 2u\theta\cos\theta} - \int 1/(4u^2)\cos 2u\theta(-\sin\theta)\, d\theta]$

$= [4u^2/(4u^2-1)][1/(2u)\sin 2u\theta\sin\theta + 1/(4u^2)\cos 2u\theta\cos\theta]$

$= -1/2\cos\theta + 2u^2/(4u^2 - 1)[1/(2u)\sin 2u\theta\sin\theta + 1/(4u^2)\cos 2u\theta\cos\theta]$

$= -1/2\cos\theta + s_2[u \sin 2u\theta\sin\theta + (1/2)\cos 2u\theta\cos\theta].$

62 $\int \cos^3 u\theta \sin\theta\, d\theta \; \{= \underline{-\cos^3 u\theta\cos\theta} + {}^A\int -3u\cos^2 u\theta \sin u\theta\cos\theta\, d\theta\}$

$A\{\int -3u\cos^2 u\theta\sin u\theta\cos\theta\, d\theta = \underline{-3u\cos^2 u\theta \sin u\theta\sin\theta} - {}^B\int 6u^2\cos u\theta\sin^2 u\theta\sin\theta - 3u^2\cos^3 u\theta\sin\theta\, d\theta\}$

$B\{-\int 6u^2\cos u\theta\sin^2 u\theta\sin\theta - 3u^2\cos^3 u\theta\sin\theta\, d\theta = {}^C - \int 6u^2\cos u\theta\sin\theta\, d\theta + \int 9u^2\cos^3 u\theta\sin\theta\, d\theta\}$

$C\{-\int 6u^2\cos u\theta\sin\theta\, d\theta = -\underline{6u^4/(u^2-1)\,[1/u\sin u\theta\sin\theta + 1/u^2\cos u\theta\cos\theta]}\}$

$= [1/(1-9u^2)][-\cos^3 u\theta\cos\theta - 3u\cos^2 u\theta\sin u\theta\sin\theta] + [1/(1-9u^2)][-6u^4/(u^2-1)][1/u\sin u\theta\sin\theta + 1/u^2\cos u\theta\cos\theta].$

$= -r_3[\cos^3 u\theta\cos\theta + 3u\cos^2 u\theta\sin u\theta\sin\theta] - 6r_3r_2[u^3\sin u\theta\sin\theta + u^2\cos u\theta\cos\theta].$

63 $\int \cos^4 u\theta\sin\theta\, d\theta \; \{= \underline{-\cos^4 u\theta\cos\theta} - {}^A\int -4u\cos^3 u\theta(-\sin u\theta)\cos\theta\, d\theta\}$

$A\{-\int 4u\cos^3 u\theta\sin u\theta\cos\theta\, d\theta = \underline{-4u\cos^3 u\theta\sin u\theta\sin\theta} - {}^B\int[-12u^2\cos^2 u\theta(-\sin^2 u\theta) - 4u^2\cos^4 u\theta]\sin\theta\, d\theta\}$

$B\{-\int[-12u^2\cos^2 u\theta(-\sin^2 u\theta) - 4u^2\cos^4 u\theta]\sin\theta\, d\theta = {}^C - \int 12u^2\cos^2 u\theta\sin\theta\, d\theta + \underline{\int 16u^2\cos^4 u\theta\sin\theta\, d\theta}\}$

$C\{-\int 12u^2\cos^2 u\theta\sin\theta\, d\theta = \underline{6u^2\cos\theta - 24u^4/(4u^2-1)[1/(2u)\sin 2u\theta\sin\theta + 1/(4u^2)\cos 2u\theta\cos\theta]}\}$

$= [1/(1-16u^2)][-\cos^4 u\theta\cos\theta - 4u\cos^3 u\theta\sin u\theta\sin\theta + 6u^2\cos\theta]$

$+ [1/(1-16u^2)][-24u^4/(4u^2-1)][1/(2u)\sin 2u\theta\sin\theta + 1/(4u^2)\cos 2u\theta\cos\theta].$

$= r_4[-\cos^4 u\theta\cos\theta - 4u\cos^3 u\theta\sin u\theta\sin\theta + 6u^2\cos\theta] - r_4 s_2[12u^3\sin 2u\theta\sin\theta + 6u^2\cos 2u\theta\cos\theta].$

64 $\int \cos^5 u\theta\sin\theta\, d\theta \; \{= \underline{-\cos^5 u\theta\cos\theta} + {}^A\int 5u\cos^4 u\theta(-\sin u\theta)\cos\theta\, d\theta\}$

$A\{\int 5u\cos^4 u\theta(-\sin u\theta)\cos\theta\, d\theta = \underline{-\sin\theta[5u\cos^4 u\theta\sin u\theta} - {}^B\int[20u^2\cos^3 u\theta(-\sin^2 u\theta) + 5u^2\cos^5 u\theta](-\sin\theta)\, d\theta\}$

$B\{\int[20u^2\cos^3 u\theta(-1 + \cos^2 u\theta) + 5u^2\cos^5 u\theta](\sin\theta)\, d\theta = -{}^C\int 20u^2\cos^3 u\theta\sin\theta\, d\theta + \underline{25u^2\int\cos^5 u\theta\sin\theta\, d\theta}\}$

$C\{-\int 20u^2\cos^3 u\theta\sin\theta\, d\theta = \underline{[20u^2/(1-9u^2)][\cos^3 u\theta\cos\theta + 3u\cos^2 u\theta\sin u\theta\sin\theta]}$

$+ \underline{[20u^2/(1-9u^2)][6u^4/(u^2-1)][1/u\sin u\theta\sin\theta + 1/u^2\cos u\theta\cos\theta]}.\}$

$= r_5[-\cos^5 u\theta\cos\theta - 5u\cos^4 u\theta\sin u\theta\sin\theta] + 20u^2 r_5 r_3[\cos^3 u\theta\cos\theta + 3u\cos^2 u\theta\sin u\theta\sin\theta]$

$+ 120u^6 r_5 r_3 r_2[1/u\sin u\theta\sin\theta + 1/u^2\cos u\theta\cos\theta].$

65 $\int \cos^6 u\theta\sin\theta\, d\theta \; \{= \underline{-\cos\theta\cos^6 u\theta} + {}^A\int -6u\cos^5 u\theta\sin u\theta\cos\theta\, d\theta\}$

$A\{\int -6u\cos^5 u\theta\sin u\theta\cos\theta\, d\theta = \underline{-6u\cos^5 u\theta\sin u\theta\sin\theta} + {}^B\int[30u^2\cos^4 u\theta(-\sin^2 u\theta) + 6u^2\cos^6 u\theta]\sin\theta\, d\theta\}$

$B\{\int[30u^2\cos^4 u\theta(-\sin^2 u\theta) + 6u^2\cos^6 u\theta]\sin\theta\, d\theta = {}^C\int -30u^2\cos^4 u\theta\sin\theta\, d\theta + \underline{36u^2\int\cos^6 u\theta\sin\theta\, d\theta}\}$

$C\{\int -30u^2\cos^4 u\theta\sin\theta\, d\theta = \underline{-[30u^2/(1-16u^2)][-\cos^4 u\theta\cos\theta - 4u\cos^3 u\theta\sin u\theta\sin\theta + 6u^2\cos\theta]}$

$- \underline{[30u^2/(1-16u^2)][-24u^4/(4u^2-1)][1/2u\sin 2u\theta\sin\theta + 1/(4u^2)\cos 2u\theta\cos\theta]}.\}$

$= r_6[-\cos\theta\cos^6 u\theta - 6u\cos^5 u\theta\sin u\theta\sin\theta] - 30u^2 r_6 r_4[-\cos^4 u\theta\cos\theta - 4u\cos^3 u\theta\sin u\theta\sin\theta + 6u^2\cos\theta]$

$+ 720u^6 r_6 r_4 s_2[1/(2u)\sin 2u\theta\sin\theta + 1/(4u^2)\cos 2u\theta\cos\theta].$

66 $\int \cos^7 u\theta\sin\theta\, d\theta \; \{= -\underline{\cos^7 u\theta\cos\theta} + {}^A\int 7u\cos^6 u\theta(-\sin u\theta)\cos\theta\, d\theta\}$

$A\{\int -7u\cos^6 u\theta\sin u\theta\cos\theta\, d\theta = \underline{-7u\cos^6 u\theta\sin u\theta\sin\theta} + {}^B\int 42u^2\cos^5 u\theta(-\sin^2 u\theta)\sin\theta\, d\theta + \underline{\int 7u^2\cos^7 u\theta\sin\theta\, d\theta}\}$

$B\{\int -42u^2\cos^5 u\theta\sin^2 u\theta\sin\theta\, d\theta = \int -42u^2\cos^5 u\theta(1 - \cos^2 u\theta)\sin\theta\, d\theta = {}^C\int -42u^2\cos^5 u\theta\sin\theta\, d\theta + \underline{\int 42u^2\cos^7 u\theta\sin\theta\, d\theta}\}$

$C\{\int -42u^2\cos^5 u\theta\sin\theta\, d\theta = \underline{-42u^2 r_5[-\cos^5 u\theta\cos\theta - 5u\cos^4 u\theta\sin u\theta\sin\theta]}$

$\underline{-840u^4 r_5 r_3[\cos^3 u\theta\cos\theta + 3u\cos^2 u\theta\sin u\theta\sin\theta] - 5040u^8 r_5 r_3 r_2[1/u\sin u\theta\sin\theta + 1/u^2\cos u\theta\cos\theta]}.$

$= -r_7\cos^7 u\theta\cos\theta - 7u\,r_7\cos^6 u\theta\sin u\theta\sin\theta - 42u^2 r_7 r_5[-\cos^5 u\theta\cos\theta - 5u\cos^4 u\theta\sin u\theta\sin\theta]$

$- 840u^4 r_7 r_5 r_3[\cos^3 u\theta\cos\theta + 3u\cos^2 u\theta\sin u\theta\sin\theta] - 5040u^8 r_7 r_5 r_3 r_2[1/u\sin u\theta\sin\theta + 1/u^2\cos u\theta\cos\theta].$

$\int \cos^n u\theta \cos\theta \, d\theta$

$r_0 = 1/u$, $r_1 = 1/u^2$, $r_2 = 1/(u^2 - 1)$, $r_3 = 1/(1 - 9u^2)$, $r_4 = 1/(1 - 16u^2)$, $r_5 = 1/(1 - 25u^2)$,
$r_6 = 1/(1 - 36u^2)$, $r_7 = 1/(1 - 49u_2)$, $0 = 1/(2u)$, $s_1 = 1/(4u^2)$, and $s_2 = 1/(4u^2 - 1)$.

67 $\int \cos u\theta\cos\theta\, d\theta = \underline{\cos u\theta\sin\theta} - [\int -u\sin u\theta\sin\theta\, d\theta = u\sin u\theta\cos\theta - \int u^2\cos u\theta\cos\theta\, d\theta]$

$= [1/(1-u^2)][\cos u\theta\sin\theta - u\sin u\theta\cos\theta] = -r_2[\cos u\theta\sin\theta - u\sin u\theta\cos\theta].$

68 $\int \cos^2 u\theta\cos\theta\, d\theta = \int[1/2 + 1/2\cos 2u\theta]\cos\theta\, d\theta = 1/2\sin\theta + 1/2\int\cos 2u\theta\cos\theta\, d\theta$

$[\int\cos 2u\theta\cos\theta = \underline{\cos 2u\theta\sin\theta} - [\int(-2u)\sin 2u\theta\sin\theta\, d\theta = \underline{2u\sin 2u\theta\cos\theta} - \int 4u^2\cos 2u\theta\cos\theta\, d\theta] = [1/(1-4u^2)][\cos 2u\theta\sin\theta - 2u\sin 2u\theta\cos\theta]$

$= 1/2\sin\theta - 1/2\,s_2[\cos 2u\theta\sin\theta - 2u\sin 2u\theta\cos\theta].$

69 $\int COS^3u\theta\, COS\theta\, d\theta$ $\{ = \underline{COS^3u\theta\, SIN\theta} - {}^A\int -3u\, COS^2u\theta\, SINu\theta\, SIN\theta\, d\theta\}$

$A\{\int 3u\, COS^2u\theta\, SINu\theta\, SIN\theta\, d\theta = \underline{-3u\, COS^2u\theta\, SINu\theta\, COS\theta} + {}^B\int 6u^2\, COSu\theta\, (-SIN^2u\theta)\, COS\theta\, d\theta + \underline{\int 3u^2\, COS^3u\theta\, COS\theta\, d\theta}\}$

$B\{\int -6u^2\, COSu\theta\, SIN^2u\theta\, COS\theta\, d\theta = \int 6u^2\, COSu\theta\, (-1 + COS^2u\theta)\, COS\theta\, d\theta = {}^C\int -6u^2\, COSu\theta\, COS\theta\, d\theta + \underline{\int 6u^2\, COS^3u\theta\, COS\theta\, d\theta}\}$

$C\{\int -6u^2\, COSu\theta\, COS\theta\, d\theta = \underline{[-6u^2/(1-u^2)]\,[COSu\theta\, SIN\theta - u\, SINu\theta\, COS\theta]}\}$

$= [1/(1-9u^2)]\,[COS^3u\theta\, SIN\theta - 3u\, COS^2u\theta\, SINu\theta\, COS\theta] + [1/(1-9u^2)]\,[-6u^2/(1-u^2)]\,[COSu\theta\, SIN\theta - u\, SINu\theta\, COS\theta].$

$$= r_3 \left[COS^3u\theta\, SIN\theta - 3u\, COS^2u\theta\, SINu\theta\, COS\theta \right] + 6u^2 r_3 r_2 \left[COSu\theta\, SIN\theta - u\, SINu\theta\, COS\theta \right]$$

70 $\int COS^4u\theta\, COS\theta\, d\theta$ $\{ = \underline{COS^4u\theta\, SIN\theta} - {}^A\int 4u\, COS^3u\theta\,(-SINu\theta)\, SIN\theta\, d\theta\}$

$A\{\int 4u\, COS^3u\theta\, SINu\theta\, SIN\theta\, d\theta = \underline{-4u\, COS^3u\theta\, SINu\theta\, COS\theta} + {}^B\int 12u^2\, COS^2u\theta\,(-SIN^2u\theta)\, COS\theta\, d\theta + \underline{\int 4u^2\, COS^4u\theta\, COS\theta\, d\theta}\}$

$B\{\int 12u^2\, COS^2u\theta\,(-SIN^2u\theta)\, COS\theta\, d\theta = {}^C\int -12u^2\, COS^2u\theta\, COS\theta\, d\theta + \underline{\int 12u^2\, COS^4u\theta\, COS\theta\, d\theta}\}$

$C\{\int -12u^2\, COS^2u\theta\, COS\theta\, d\theta = \underline{-6u^2\, SIN\theta + [-6u^2/(1-4u^2)]\,[COS2u\theta\, SIN\theta - 2u\, SIN2u\theta\, COS\theta]}\}$

$= [1/(1-16u^2)]\,[COS^4u\theta\, SIN\theta - 4u\, COS^3u\theta\, SINu\theta\, COS\theta - 6u^2\, SIN\theta]$
$+ [1/(1-16u^2)]\,[-6u^2/(1-4u^2)]\,[COS2u\theta\, SIN\theta - 2u\, SIN2u\theta\, COS\theta].$

$$= r_4 \left[COS^4u\theta\, SIN\theta - 4u\, COS^3u\theta\, SINu\theta\, COS\theta - 6u^2\, SIN\theta \right] + 6u^2 r_4 s_2 \left[COS2u\theta\, SIN\theta - 2u\, SIN2u\theta\, COS\theta \right].$$

71 $\int COS^5u\theta\, COS\theta\, d\theta$ $\{ = \underline{COS^5u\theta\, SIN\theta} - {}^A\int 5u\, COS^4u\theta\,(-SINu\theta)\, SIN\theta\, d\theta\}$

$A\{\int 5u\, COS^4u\theta\, SINu\theta\, SIN\theta\, d\theta = \underline{-5u\, COS^4u\theta\, SINu\theta\, COS\theta} + {}^B\int -20u^2\, COS^3u\theta\, SIN^2u\theta\, COS\theta\, d\theta + \underline{\int 5u^2\, COS^5u\theta\, COS\, d\theta}\}$

$B\{\int -20u^2\, COS^3u\theta\, SIN^2u\theta\, COS\theta\, d\theta = -20u^2\int COS^3u\theta\,(1-COS^2u\theta)\, COS\theta\, d\theta = {}^C\int -20u^2\, COS^3u\theta\, COS\theta\, d\theta + \underline{20u^2\int COS^5u\theta\, COS\theta\, d\theta}\}$

$C\{\int -20u^2\, COS^3u\theta\, COS\theta\, d\theta = \underline{[-20u^2/(1-9u^2)]\,[COS^3u\theta\, SIN\theta - 3u\, COS^2u\theta\, SINu\theta\, COS\theta]}$
$\underline{[-20u^2/(1-9u^2)]\,[-6u^2/(1-u^2)]\,[COSu\theta\, SIN\theta - u\, SINu\theta\, COS\theta]}\}$

$= [1/(1-25u^2)]\,[COS^5u\theta\, SIN\theta - 5u\, COS^4u\theta\, SINu\theta\, COS\theta] + [1/(1-25u^2)]\,[-20u^2/(1-9u^2)]\,[COS^3u\theta\, SIN\theta - 3u\, COS^2u\theta\, SINu\theta\, COS\theta]$
$+ [1/(1-25u^2)]\,[-20u^2/(1-9u^2)]\,[-6u^2/(1-u^2)]\,[COSu\theta\, SIN\theta - u\, SINu\theta\, COS\theta].$

$$= r_5 \left[COS^5u\theta\, SIN\theta - 5u\, COS^4u\theta\, SINu\theta\, COS\theta \right] - 20u^2 r_3 r_5 \left[COS^3u\theta\, SIN\theta - 3u\, COS^2u\theta\, SINu\theta\, COS\theta \right]$$
$$- 120u^4 r_3 r_5 r_2 \left[COSu\theta\, SIN\theta - u\, SINu\theta\, COS\theta \right].$$

72 $\int COS^6u\theta\, COS\theta\, d\theta$ $\{ = \underline{COS^6u\theta\, SIN\theta} - {}^A\int -6u\, SINu\theta\, COS^5u\theta\, SIN\theta\, d\theta\}$

$A\{-\int -6u\, SINu\theta\, COS^5u\theta\, SIN\theta\, d\theta = \underline{-6u\, SINu\theta\, COS^5u\theta\, COS\theta} + {}^B\int -30u^2\, SIN^2u\theta\, COS^4u\theta\, COS\, d\theta + \underline{6u^2\int COS^6u\theta\, COS\theta\, d\theta}\}$

$B\{\int -30u^2\,(1-COS^2u\theta)\, COS^4u\theta\, COS\, d\theta = {}^C\int -30u^2\, COS^4u\theta\, COS\, d\theta + \underline{\int 30u^2\, COS^6u\theta\, COS\, d\theta}\}$

$C\{\int -30u^2\, COS^4u\theta\, COS\, d\theta = \underline{[-30u^2/(1-16u^2)]\,[COS^4u\theta\, SIN\theta - 4u\, COS^3u\theta\, SINu\theta\, COS\theta - 6u^2\, SIN\theta]}$
$+ \underline{[-30u^2/(1-16u^2)]\,[-6u^2/(1-4u^2)]\,[COS2u\theta\, SIN\theta - 2u\, SIN2u\theta\, COS\theta]}\}.$

$= [1/(1-36u^2)]\,[COS^6u\theta\, SIN\theta - 6u\, SINu\theta\, COS^5u\theta\, COS\theta] +$
$1/(1-36u^2)]\,[-30u^2/(1-16u^2)]\,[COS^4u\theta\, SIN\theta - 4u\, COS^3u\theta\, SINu\theta\, COS\theta - 6u^2\, SIN\theta]$
$+ [1/(1-36u^2)]\,[-30u^2/(1-16u^2)]\,[-6u^2/(1-4u^2)]\,[COS2u\theta\, SIN\theta - 2u\, SIN2u\theta\, COS\theta]$

$$= r_6 \left[COS^6u\theta\, SIN\theta - 6u\, SINu\theta\, COS^5u\theta\, COS\theta \right] - 30u^2 r_6 r_4 \left[COS^4u\theta\, SIN\theta - 4u\, COS^3u\theta\, SINu\theta\, COS\theta - 6u^2\, SIN\theta \right]$$
$$- 180u^4 r_6 r_4 s_2 \left[COS2u\theta\, SIN\theta - 2u\, SIN2u\theta\, COS\theta \right].$$

73 $\int COS^7u\theta\, COS\theta\, d\theta$ $= \underline{COS^7u\theta\, SIN\theta} - {}^A\int 7u\, COS^6u\theta\,(-SINu\theta)\, SIN\theta\, d\theta$

$A\{-\int 7u\, COS^6u\theta\,(-SINu\theta)\, SIN\theta\, d\theta = \underline{-7u\, COS^6u\theta\, SINu\theta\, COS\theta} + {}^B\int -42u^2\, COS^5u\theta\, SIN^2u\theta\, COS\theta\, d\theta + \underline{\int 7u^2\, COS^7u\theta)\, COS\theta\, d\theta}$

$B\{\int -42u^2\, COS^5u\theta\, SIN^2u\theta\, COS\theta\, d\theta = {}^C\int -42u^2\, COS^5u\theta\, COS\theta\, d\theta + \underline{\int 42u^2\, COS^7u\theta\, COS\theta\, d\theta}$

$C\{\int -42u^2\, COS^5u\theta\, COS\theta\, d\theta = -42u^2 r_5\,[COS^5u\theta\, SIN\theta - 5u\, COS^4u\theta\, SINu\theta\, COS\theta] + 840u^4 r_3 r_5\,[COS^3u\theta\, SIN\theta - 3u\, COS^2u\theta\, SINu\theta\, COS\theta]$
$+ \underline{5040\, u^6 r_3 r_5 r_2\,[COSu\theta\, SIN\theta - u\, SINu\theta\, COS\theta}].$

$$= r_7\, COS^7u\theta\, SIN\theta - 7u\, r_7\, COS^6u\theta\, SINu\theta\, COS\theta - 42u^2 r_7 r_5 \left[COS^5u\theta\, SIN\theta - 5u\, COS^4u\theta\, SINu\theta\, COS\theta \right]$$
$$+ 840u^4 r_7 r_5 r_3 \left[COS^3u\theta\, SIN\theta - 3u\, COS^2u\theta\, SINu\theta\, COS\theta \right] + 5040u^6 r_7 r_5 r_3 r_2 \left[COSu\theta\, SIN\theta - u\, SINu\theta\, COS\theta \right].$$

$\int \sigma^n\, COS\,(B - \sigma)\, d\sigma$

74 $\int \sigma\, COS\,(B-\sigma)\, d\sigma$ $= -\sigma\, SIN\,(B-\sigma) + [\int SIN\,(B-\sigma)\, d\sigma = COS\,(B-\sigma)] = -\sigma\, SIN\,(B-\sigma) + COS\,(B-\sigma).$

75 $\int \sigma^2\, COS\,(B-\sigma)\, d\sigma$ $= -\sigma^2\, SIN\,(B-\sigma) + \{\int 2\sigma\, SIN\,(B-\sigma)\, d\sigma = 2\sigma\, COS\,(B-\sigma) + [-\int 2\, COS\,(B-\sigma)\, d\sigma = 2\, SIN\,(B-\sigma)]\}$

$= -\sigma^2\, SIN\,(B-\sigma) + 2\sigma\, COS\,(B-\sigma) + 2\, SIN\,(B-\sigma).$

76 $\int \sigma^3\, COS\,(B-\sigma)\, d\sigma$ $= -\sigma^3\, SIN\,(B-\sigma)$

$+ \{\int 3\sigma 2\, SIN\,(B-\sigma)\, d\sigma = 3\sigma^2\, COS\,(B-\sigma) + [-\int 6\sigma\, COS\,(B-\sigma)\, d\sigma = 6\sigma\, SIN\,(B-\sigma) + [-\int 6\, SIN\,(B-\sigma)\, d\sigma = -6\, COS\,(B-\sigma)]]\}$

$= -\sigma^3\, SIN\,(B-\sigma) + 3\sigma^2\, COS\,(B-\sigma) + 6\sigma\, SIN\,(B-\sigma) - 6\, COS\,(B-\sigma)$

$\int \partial\, [\, f(x,y)\, dx\,]/\partial y \;=\; \partial\, [\, F(x,y)\,]/\partial y + c$, (see [29]Rektorys, Karel)

Where $\int f(x,y)\, dx = F(x,a)$ with $f(x,y)$ and $\partial f(x,y)/\partial y$, are real and continuous, then
$\int \partial\, [\, f(x,y)\, dx\,]/\partial y = \partial\, [\, F(x,y)\,]/\partial y + c$; e.g. having the solution for $\int d\theta/(A + B\, COS\theta)$, it follows that :
$\int d\theta/(A + B\, COS\theta)^2 = -\int \partial\, [\, d\theta/(A + B\, COS\theta)^2\,]/\partial y$
$= -\partial\, [\, 2\, [A^2 - B^2]^{1/2} / [A^2 - B^2]\,]\, TAN^{-1}\,(\, [\, (A-B)/(A+B)\,]^{1/2}\, TAN\,(\theta/2)\,)/\partial y.$

$\int COS\theta\, d\theta/\, [\, A + B\, COS\,\theta\,]^n$ (see [29]Rektorys, Karel)

1) $COS\,\theta = 2\, COS^2(\theta/2) - 1$
2) $SIN\,\theta = 2\, SIN(\theta/2)\, COS(\theta/2)$.

SUBSTITUTING 1 AND 2 ABOVE INTO THE IDENTITY $1 - SIN^2\theta = COS^2\theta$,

$COS^2\theta = 1 - SIN^2\theta = 1 - 4\, SIN^2(\theta/2)\, COS^2(\theta/2) = 1 - 4\,(1 - COS^2(\theta/2))\, COS^2(\theta/2) = 1 - 4\, COS^2(\theta/2) + 4\, COS^4(\theta/2)$

DIVIDING $1 - 4\, SIN^2(\theta/2)\, COS^2(\theta/2) = 1 - 4\, COS^2(\theta/2) + 4\, COS^4(\theta/2)$ BY $4\, COS^4(\theta/2)$

$1 + TAN^2(\theta/2) = 1/COS^2(\theta/2)$ AND $COS^2(\theta/2) = 1/(TAN^2(\theta/2) + 1)$

WITH $\mathrm{TAN}(\theta/2) = Z$, $\mathrm{COS}^2(\theta/2) = 1/(\mathrm{TAN}^2(\theta/2)+1) = 1/(Z^2+1)$

WITH $\mathrm{TAN}(\theta/2) = Z = [\mathrm{SIN}^2(\theta/2)/\mathrm{COS}^2(\theta/2)]^{1/2} = [1/2(1-\mathrm{COS}\theta)]^{1/2}/[1/2(1+\mathrm{COS}\theta)]^{1/2} = \mathrm{SIN}\,\theta/(1+\mathrm{COS}\,\theta)$

$\mathrm{COS}(\theta) = 2\,\mathrm{COS}^2(\theta/2) - 1 = 2/(Z^2+1) - 1 = (1-Z^2)/(Z^2+1)$

$\mathrm{SIN}(\theta) = [1-\mathrm{COS}^2(\theta)]^{1/2} = \left[((Z^2+1)^2-(1-Z^2)^2)/(Z^2+1)^2\right]^{1/2} = 2Z/(Z^2+1)$

$d\,\mathrm{SIN}\theta/d\theta = d(2Z/(Z^2+1))/dZ \cdot dZ/d\theta = 2(1-Z^2)/(Z^2+1)^2\,dZ/d\theta = \mathrm{COS}\theta$

$\therefore\ 2(1-Z^2)/(Z^2+1)^2\,dZ/d\theta = (1-Z^2)/(Z^2+1)$, and $d\theta = 2\,dZ/(1+Z^2)$

therefore the substitutions for $\int f(\mathrm{COS}\theta,\mathrm{SIN}\theta)\,d\theta$ below are permitted.

$Z = \mathrm{TAN}(\theta/2)$, $\mathrm{COS}\theta = (1-Z^2)/(Z^2+1)$, $\mathrm{SIN}\theta = 2Z/(Z^2+1)$ and $d\theta = 2\,dZ/(Z^2+1)$

77&78 $\int \mathrm{COS}\theta\,d\theta/[A+B\,\mathrm{COS}\theta] = 1/B\int d\theta - A/B\int d\theta/[A+B\,\mathrm{COS}\theta] = \theta/B - A/B\int d\theta/[A+B\,\mathrm{COS}\theta]$

WITH $Z = \mathrm{TAN}(\theta/2)$: $d\theta = 2dZ/(Z^2+1)$, $\mathrm{SIN}\theta = 2Z/(Z^2+1)$, $\mathrm{COS}\theta = (1-Z^2)/(Z^2+1)$

$\int d\theta/[A+B\,\mathrm{COS}\theta] = \int[2\,dZ/(Z^2+1)]/[A+B(1-Z^2)/(Z^2+1)] = \int 2\,dZ/[(A+B)+(A-B)\cdot Z^2]$

WITH $[(A+B)/(A-B)]^{1/2}U = Z$, $[(A+B)/(A-B)]^{1/2}dU = dZ$,

$2[(A+B)/(A-B)]^{1/2}\int dU/[(A+B)(1+U^2)] = 2[A^2-B^2]^{1/2}/[A^2-B^2]\int dU/[1+U^2]$.

WITH $U = \mathrm{TAN}\mu$ AND $dU = \mathrm{SEC}^2\mu\,d\mu$ and $X = 2[A^2-B^2]^{1/2}/[A^2-B^2]$,

$X\int dU/[1+U^2] = X\int \mathrm{SEC}^2\mu\,d\mu/\mathrm{SEC}^2\mu = X\mu = X\,\mathrm{TAN}^{-1}U$

77 $\int d\theta/[A+B\,\mathrm{COS}\theta] = [2[A^2-B^2]^{1/2}/[A^2-B^2]]\,\mathrm{TAN}^{-1}\left([(A-B)/(A+B)]^{1/2}\,\mathrm{TAN}(\theta/2)\right)$.

78 $\int \mathrm{COS}\theta\,d\theta/[A+B\,\mathrm{COS}\theta] = \int \theta/B\,d\theta - \int(A/B)\,d\theta/[A+B\,\mathrm{COS}\theta]$

$= \theta^2/(2B) - A/B\left[2[A^2-B^2]^{1/2}/[A^2-B^2]\right]\mathrm{TAN}^{-1}\left([(A-B)/(A+B)]^{1/2}\,\mathrm{TAN}(\theta/2)\right) + C.$

79 $\int \mathrm{COS}\theta\,d\theta/[A+B\,\mathrm{COS}\theta]^2$

WITH $Z = \mathrm{TAN}(\theta/2)$: $d\theta = 2dZ/(Z^2+1)$, $\mathrm{SIN}\theta = 2Z/((Z^2+1)$, $\mathrm{COS}\theta = (1-Z^2)/((Z^2+1)$

$\int 2(1-Z^2)dZ/[(A+B)+(A-B)Z^2]^2$

WITH $Z = [(A+B)/(A-B)]^{1/2}\mu$, $X = [(A-B)/(A+B)]^{1/2}$, $\mu = X\cdot Z$, $1/X\,d\mu = dZ$

$\int 2/X[1-\mu^2/X^2]d\mu/[(A+B)^2(1+\mu^2)^2] = 2/[X(A+B)^2]\left[{}^{A}\int d\mu/(1+\mu^2)^2\right] - {}^{B}\int \mu^2/X^2 d\mu/(1+\mu^2)^2]$

$\mu = \mathrm{TAN}\phi$, $d\mu = \mathrm{SEC}^2\phi\,d\phi$, $\mathrm{COS}\phi = 1/(1+X^2Z^2)^{1/2}$, $\mathrm{SIN}\phi = XZ/(1+X^2Z^2)^{1/2}$

$A\{\int d\mu/(1+\mu^2)^2 = \int \mathrm{SEC}^2\phi\,d\phi/[1+\mathrm{TAN}^2\phi]^2 = \int \mathrm{COS}^2\phi\,d\phi = 1/2[\phi + \mathrm{COS}\phi\,\mathrm{SIN}\phi]$

$2/[X(A+B)^2]\int d\mu/(1+\mu^2)^2 = 1/[X(A+B)^2][{}^{C}\mathrm{ATN}(XZ) + {}^{D}XZ/(1+X^2Z^2)]\}$

$B\{\int \mu^2 d\mu/(1+\mu^2)^2 = \int \mathrm{SEC}^2\phi\,\mathrm{TAN}^2\phi\,d\phi/[1+\mathrm{TAN}^2\phi]^2 = \int \mathrm{SIN}^2\phi\,d\phi = 1/2[\phi - \mathrm{COS}\phi\,\mathrm{SIN}\phi]$

$-2/[X^3(A+B)^2]\int \mu^2 d\mu/(1+\mu^2)^2 = -1/[X^3(A+B)^2][{}^{E}\mathrm{ATN}(XZ) - {}^{F}XZ/(1+X^2Z^2)]\}$

Using $\mathrm{TAN}\,\delta/2 = \left([1/2(1-\mathrm{COS}\delta)]/[1/2(1+\mathrm{COS}\delta)]\right)^{1/2} = \mathrm{SIN}\delta/(1+\mathrm{COS}\delta) = (1-\mathrm{COS}\delta)/\mathrm{SIN}\delta$

$[Z/(1+X^2Z^2)] = (1-\mathrm{COS}\theta)\mathrm{SIN}\theta/[\mathrm{SIN}^2\theta + X^2(1-\mathrm{COS}\theta)^2] = \mathrm{SIN}\theta/[(1+\mathrm{COS}\theta)+X^2(1-\mathrm{COS}\theta)] = 1/2(A+B)\mathrm{SIN}\theta/(A+B\,\mathrm{COS}\theta)$

Combining the coefficients of C with E and D with F and simplifying:

79 $\int \mathrm{COS}\theta\,d\theta/[A+B\,\mathrm{COS}\theta]^2 =$

$A\cdot\mathrm{SIN}\theta/[(A^2-B^2)(A+B\,\mathrm{COS}\theta)] - [2B/(A^2-B^2)^{3/2}]\mathrm{ATN}\left([(A-B)/(A^2-B^2)^{1/2}]\,\mathrm{TAN}(\theta/2)\right)$.

80 $\int \mathrm{COS}\theta\,d\theta/[A+B\,\mathrm{COS}\theta]^3$

$\int 2(1-Z^2)(1+Z^2)dZ/[(A+B)+(A-B)Z^2]^3 = 2/(A+B)^3\int(1-Z^4)dZ/[1+(A-B)Z^2/(A+B)]^3$

WITH $Z = [(A+B)/(A-B)]^{1/2}\mu$, $X = [(A-B)/(A+B)]^{1/2}$, $\mu = X\cdot Z$, $1/X\,d\mu = dZ$

$2/(A+B)^3\int[1-\mu^4/X^4]1/X\,d\mu/[1+\mu^2]^3$, WITH $\mu = \mathrm{TAN}\phi$, $d\mu = \mathrm{SEC}^2\phi\,d\phi$

$2/[X(A+B)^3]\int \mathrm{SEC}^2\phi\,d\phi/\mathrm{SEC}^6\phi - 2/[X^5(A+B)^3]\int \mathrm{TAN}^4\phi\,\mathrm{SEC}^2\phi\,d\phi/\mathrm{SEC}^6\phi$

$\int \mathrm{COS}\theta\,d\theta/[A+B\,\mathrm{COS}\theta]^3 = 2/[X(A+B)^3]\int \mathrm{COS}^4\phi\,d\phi - 2/[X^5(A+B)^3]\int \mathrm{SIN}^4\phi\,d\phi$

DEFINING CONSTANT $J = 2/[X(A+B)^3]$ and $K = -2/[X^5(A+B)^3]$

Using and above for $\int \mathrm{COS}^4\phi\,d\phi$ and $\int \mathrm{SIN}^4\phi\,d\phi$.

$\int \mathrm{COS}^4\phi\,d\phi = 3/8\,\phi + 1/4\,\mathrm{SIN}\,2\phi + 1/32\,\mathrm{SIN}\,4\phi$, $\int \mathrm{SIN}^4\phi\,d\phi = \underline{3/8\,\phi - 1/4\,\mathrm{SIN}2\phi + 1/32\,\mathrm{SIN}4\phi}$.

$Z = \mathrm{TAN}(\delta/2) = \mathrm{SIN}\delta/(1+\mathrm{COS}\delta) = (1-\mathrm{COS}\delta)/\mathrm{SIN}\delta$

$\int \mathrm{COS}\theta\,d\theta/[A+B\,\mathrm{COS}\theta]^3 = 3/8[J+K]\phi + 1/4[J-K]\mathrm{SIN}2\phi + 1/32[J+K]\mathrm{SIN}4\phi$

$= {}^{M}3/8[J+K]\phi + {}^{N}1/2[J-K]\mathrm{SIN}\phi\,\mathrm{COS}\phi + {}^{O}1/32[J+K][\mathrm{SIN}\phi\,\mathrm{COS}^3\phi - \mathrm{SIN}^3\phi\,\mathrm{COS}\phi]$

With $\mathrm{TAN}\phi = \mu = XZ$, $\mathrm{SIN}\phi = XZ/(1+X^2Z^2)^{1/2}$, $\mathrm{COS}\phi = 1/(1+X^2Z^2)^{1/2}$, and

$Z = \mathrm{TAN}(\delta/2) = \mathrm{SIN}\delta/(1+\mathrm{COS}\delta) = (1-\mathrm{COS}\delta)/\mathrm{SIN}\delta$

$M\{3/8[J+K]\phi = 3/8[J+K]\mathrm{ATN}\left([(A-B)/(A^2-B^2)^{1/2}]\,\mathrm{TAN}(\theta/2)\right)\}$

$N\{1/2[J-K]\mathrm{SIN}\phi\,\mathrm{COS}\phi = 1/2[J-K]XZ/(1+X^2Z^2) = 1/2[J-K]\cdot X\cdot\mathrm{SIN}\theta/\left[(X^2+1)+(1-X^2)\mathrm{COS}\theta\right]\}$

$O\{1/32[J+K][\mathrm{SIN}\phi\,\mathrm{COS}^3\phi - \mathrm{SIN}^3\phi\,\mathrm{COS}\phi] = 1/32[J+K][XZ/(1+X^2Z^2)^2 - X^3Z^3/(1+X^2Z^2)^2]$

$= 1/32[J+K][X\cdot\mathrm{SIN}^3\theta]/\left[(1-\mathrm{COS}\theta)[(X^2+1)+(1-X^2)\mathrm{COS}\theta]^2\right]$

$-1/32[J+K][X^3\cdot\mathrm{SIN}\theta(1-\mathrm{COS}\theta)/[(X^2+1)+(1-X^2)\mathrm{COS}\theta]^2$

$= 1/32[J+K][X\cdot\mathrm{SIN}\theta][\mathrm{SIN}2\theta - X^2(1-\mathrm{COS}\theta)^2]/\left[(1-\mathrm{COS}\theta)[(X^2+1)+(1-X^2)\mathrm{COS}\theta]^2\right]\}$

80 $\int \cos\theta \, d\theta/[A + B\cos\theta]^3 = 3/8\,[J + K]\,\mathrm{ATN}\left([(A - B)/(A^2 - B^2)^{1/2}]\,\mathrm{TAN}(\theta/2) \right)$

$+ 1/2\,[J - K]\,X\,\mathrm{SIN}\theta/[(X^2 + 1) + (1 - X^2)\cos\theta]$

$+ 1/32\,[J + K][X\,\mathrm{SIN}\theta][\mathrm{SIN}2\theta - X^2(1 - \cos\theta)^2]/[(1 - \cos\theta)[(X^2 + 1) + (1 - X^2)\cos\theta]^2,$

Where $X = [(A - B)/(A + B)]^{1/2}$, $J = 2/[X(A + B)^3]$, and $K = -2/[X^5(A + B)^3]$.

81 $\int d\theta/(b + c\cos\theta)^2$

$\mathrm{SIN}\theta = 2Z/(Z^2 + 1),\ \cos\theta = (1 - Z^2)/(Z^2 + 1),\ d\theta = 2\,dz/(Z^2 + 1),\ Z = \mathrm{TAN}(\theta/2)$

$\int [2\,dz/(z^2 + 1)]/[(bz^2 + b + c - cz^2)/(z^2 + 1)]^2 = \int 2(z^2 + 1)dz/[(b + c) + (b - c)z^2]^2$

$= \int [2(z^2 + 1)/(b + c)^2]dz/[1 + (b - c)/(b + c)z^2]^2 = 2/(b + c)^2 \int (z^2 + 1)dz/[1 + (b - c)/(b + c)z^2]^2$

$\mathrm{TAN}\mu = [(b - c)/(b + c)]^{1/2} Z,\quad Z = [(b + c)/(b - c)]^{1/2}\mathrm{TAN}\mu,\quad dZ = [(b + c)/(b - c)]^{1/2}\mathrm{SEC}^2\mu\, d\mu$

$e = [(b + c)/(b - c)]^{1/2},\quad f = 2/(b + c)^2$

$f\int (e^2\,\mathrm{TAN}^2\mu + 1)e\,\mathrm{SEC}^2\mu\, d\mu/\mathrm{SEC}^4\mu = f\int e^3\,\mathrm{SIN}^2\mu + e\cos^2\mu\, d\mu = f[e^3/2(\mu - \mathrm{SIN}\mu\cos\mu) + e/2(\mu + \mathrm{SIN}\mu\cos\mu)]$

$= f(e^3/2 + e/2)\mu + f(e/2 - e^3/2)^A\,\mathrm{SIN}\mu\cos\mu$

With $\mathrm{SIN}\mu = (1/e)Z/[1 + (1/e^2)Z^2]^{1/2}$, $\cos\mu = 1/[1 + (1/e^2)Z^2]^{1/2}$, $\mu = \mathrm{ATN}[(1/e)\mathrm{TAN}(\theta/2)]$;

A $\{\ \mathrm{SIN}\mu\cos\mu = eZ/[e^2 + Z^2] = e\,\mathrm{TAN}(\theta/2)/[e^2 + \mathrm{TAN}^2(\theta/2)] = e\,\mathrm{TAN}(\theta/2)/[(e^2 - 1) + 1 + \mathrm{TAN}^2(\theta/2)]$

$= e\,\mathrm{TAN}(\theta/2)/[(e^2 - 1) + \mathrm{SEC}^2(\theta/2)] = e\,\mathrm{SIN}(\theta/2)\cos(\theta/2)/[(e^2 - 1)\cos^2(\theta/2) + 1]$

$= (1/2)e\,\mathrm{SIN}\theta/[(e^2 + 1) + (e^2 - 1)\cos\theta] = (1/2)[b^2 - c^2]^{1/2}\mathrm{SIN}\theta/[b + c\cos\theta]\}$

81 $\int d\theta/(b + c\cos\theta)^2 = 2b(b^2 - c^2)^{-3/2}\,\mathrm{ATN}[(1/e)\mathrm{TAN}(\theta/2)] - c(b^2 - c^2)^{-1}\mathrm{SIN}\theta/[b + c\cos\theta]$

82 $\int d\theta/(b + c\cos\theta)^3$

$\mathrm{SIN}\theta = 2Z/(Z^2 + 1),\ \cos\theta = (1 - Z^2)/(Z^2 + 1),\ d\theta = 2\,dz/(Z^2 + 1),\ Z = \mathrm{TAN}(\theta/2)$

$\int [2\,dz/(z^2 + 1)]/[(bz^2 + b + c - cz^2)/(z^2 + 1)]^3 = \int 2(z^2 + 1)dz/[(b + c) + (b - c)z^2]^3$

$= \int [2(z^2 + 1)/(b + c)^3]dz/[1 + (b - c)/(b + c)z^2]^3 = 2/(b + c)^3\int (z^2 + 1)\,dz/[1 + (b - c)/(b + c)z^2]^3$

$\mathrm{TAN}\mu = [(b - c)/(b + c)]^{1/2}Z,\quad Z = [(b + c)/(b - c)]^{1/2}\mathrm{TAN}\mu,\quad dZ = [(b + c)/(b - c)]^{1/2}\mathrm{SEC}^2\mu\, d\mu$

$e = [(b + c)/(b - c)]^{1/2},\quad g = 2/(b + c)^3$

$g\int (e^2\,\mathrm{TAN}^2\mu + 1)e\,\mathrm{SEC}^2\mu\, d\mu/\mathrm{SEC}^6\mu = g\int e^3\,\mathrm{SIN}^2\mu\cos^2\mu + e\cos^4\mu\, d\mu = \int g e^3\cos^2\mu + g(e - e^3)\cos^4\mu\, d\mu$

Using math appendix #2 and #4

$g\cdot e^3\int \cos^2\mu\, d\mu = g\cdot e^3[1/2\,\mu + 1/2\,\mathrm{SIN}\mu\cos\mu]$.

$g(e - e^3)\int \cos^4\mu\, d\mu = g(e - e^3)[3/8\,\mu + 3/8\,\mathrm{SIN}\mu\cos\mu + 1/4\,\mathrm{SIN}\mu\cos^3\mu]$

$[{}^A e^3g/2 + (3/8)g(e - e^3)]\mu = [4/(b + c)^3][(b + c)/(b - c)]^{1/2}\mathrm{ATN}[[(b - c)/(b + c)]^{1/2}\mathrm{TAN}(\theta/2)]$

$[e^3g/2 + (3/8)g(e - e^3)]({}^B\,\mathrm{SIN}\mu\cos\mu) = [4/(b + c)^3][(b + c)/(b - c)]^{1/2}\mathrm{SIN}\mu\cos\mu$

$[{}^C(1/4)\cdot g\cdot(e - e^3)]({}^D\,\mathrm{SIN}\mu\cos^3\mu)$

A $\{\ [(1/8)e^3g + (3/8)eg] = (1/2)(b + c)^{-3}[(b + c)/(b - c)]^{1/2}[2b - c](b - c)^{-1}$

B $\{\ \mathrm{SIN}\mu\cos\mu = Ze/[Z^2 + e^2] = e\cos^2(\theta/2)\mathrm{TAN}(\theta/2)/(1 + (e^2 - 1)\cos^2(\theta/2))$

$= e\,\mathrm{SIN}\theta/[(e^2 + 1) + (e^2 - 1)\cos\theta] = [b^2 - c^2]^{1/2}/(2b + 2c\cos\theta)\}$

C $\{\ (1/4)\cdot g\cdot(e - e^3) = -c(b - c)^{-1}(b + c)^{-3}[(b + c)/(b - c)]^{1/2}\}$

D $\{\ \mathrm{SIN}\mu\cos^3\mu = Ze^3/[Z^2 + e^2]^2 = e^3\cos^4(\theta/2)\mathrm{TAN}(\theta/2)/(1 + (e^2 - 1)\cos^2(\theta/2))^2$

$= e^3[\mathrm{SIN}\theta\cos\theta + \mathrm{SIN}\theta]/[(e^2 + 1) + (e^2 - 1)\cos\theta]^2$

$= (b + c)(b^2 - c^2)^{1/2}[\mathrm{SIN}\theta\cos\theta + \mathrm{SIN}\theta]/[2b + 2c\cos\theta]^2\}$

82 $\int d\theta/(b + c\cos\theta)^3 = (2b - c)/[2(b + c)(b^2 - c^2)^{3/2}]\mathrm{ATN}[[(b - c)/(b + c)]^{1/2}\mathrm{TAN}(\theta/2)]$

$+ (2b - c)(b + c\cos\theta)/[(b + c)^2(b - c)]$

$- c[\mathrm{SIN}\theta\cos\theta + \mathrm{SIN}\theta]/[2(b^2 - c^2)(b + c\cos\theta)^2]$

83 $\int dt = \int [12\mu W({}_{180°}L_d)/(\epsilon M^2)]\left[1/(1 - \epsilon^2) + [2\epsilon/(1 - \epsilon^2)^{3/2}]\mathrm{ATN}[(1 + \epsilon)/(1 - \epsilon^2)^{1/2}] - 1\right] d\epsilon,$

with $X = [12\mu L_d W/M^2]$,

$\int dt = X\left[\epsilon/(1 - \epsilon^2) + [2/(1 - \epsilon^2)^{3/2}]\mathrm{ATN}[(1 + \epsilon)/(1 - \epsilon^2)^{1/2}]\right]$ & $\epsilon = \mathrm{SIN}\delta$, $d\epsilon = \cos\delta\, d\delta$

${}^A\int e/(1 - e^2)\,de + {}^B\int [2/(1 - e^2)^{3/2}]\mathrm{ATN}[(1 + e)/(1 - e)]^{1/2}]\,de$

A $\{\int e/(1 - e^2)\,de = -(1/2)\,\mathrm{Ln}\,{*}1 - e^2{*}\}$

B $\{\ \int [2/(1 - e^2)^{3/2}]\mathrm{ATN}[[(1 + e)/(1 - e)]^{1/2}]de$, with $e = \mathrm{SIN}\theta$, $de = \cos\theta\, d\theta$

$= \int [2/\cos^3\theta]\mathrm{ATN}[[(1 + \mathrm{SIN}\theta)/(1 - \mathrm{SIN}\theta)]^{1/2}]\cos\theta\, d\theta$

$= 2\,\mathrm{TAN}\theta\,\mathrm{ATN}[[(1 + \mathrm{SIN}\theta)/(1 - \mathrm{SIN}\theta)]^{1/2}] - \int 2\,\mathrm{TAN}\theta^C[d(\mathrm{ATN}[[(1 + \mathrm{SIN}\theta)/(1 - \mathrm{SIN}\theta)]^{1/2}])/d\theta]\,d\theta\}$

C $\{d(\mathrm{ATN}[[(1 + \mathrm{SIN}\theta)/(1 - \mathrm{SIN}\theta)]^{1/2}])/d\theta$

$(1/[1 + (1 + \mathrm{SIN}\theta)/(1 - \mathrm{SIN}\theta)])([1/2)\cos\theta(1 - \mathrm{SIN}\theta)^{-1/2} + 1/2\cos\theta(1 + \mathrm{SIN}\theta)^{1/2}(1 - \mathrm{SIN})^{-3/2}])$

$d(\mathrm{ATN}[[(1 + \mathrm{SIN}\theta)/(1 - \mathrm{SIN}\theta)]^{1/2}])/d\theta = [(1 - \mathrm{SIN}\theta)/2][1/(1 - \mathrm{SIN}\theta)] = 1/2\}$ *[continued next page]*

$\int [2/(1-e^2)^{3/2}]\,ATN\,[\,[(1+e)/(1-e)]^{1/2}\,]\,de$

$= 2\,TAN\theta\,ATN\,(\,[(1+SIN\theta)/(1-SIN\theta)]^{1/2}\,) - [\int TAN\theta\, d\theta = -1/2\,Ln^*(1-e^2)^*\,].$

83 $\int_0^t dt = \int_{\epsilon_0}^{\epsilon_t} [\,12\mu L_d W\,(d\epsilon/dt)/(\epsilon M^2)\,]\,[\,e^2/(1-\epsilon^2) + [2\epsilon/(1-\epsilon^2)^{3/2}]\,ATN\,[\,(1+\epsilon)/(1-\epsilon^2)^{1/2}\,]\,]$

$= \,*_{\epsilon_0}^{\epsilon_t}\,[\,24\mu W/M^2\,]\,[\,e/(1-e^2)^{1/2}\,]\,ATN\,(\,[(1+e)/(1-e)]^{1/2}\,).$

APPENDIX B: POWER SERIES

MACLAURIN SERIES: **Given a continuous function y = g (x). The function is approximated by a polynomial:**

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + \ldots + a_nx^n$; i.e.

$y = g(x) \approx f_n(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + \ldots + a_nx^n.$

It's desired that the terms of the expansion decrease in absolute value as their number, n, designate increases, the polynomial is applicable to the function y's domain in the interval *x*< 1. The more closely the polynomial $f_n(x)$ approximates the function g(x) the greater the number of derivatives of the polynomial $f_n(x)$ will equal like derivatives of g(x).

Therefore the coefficients of the approximating polynomial $f_n(x)$, $a_0, a_1, a_2, \ldots a_n$, can be derived by differentiating and equating order by order of function g(x) to $f_n(x)$, at x = 0; i.e.

$f(x) = f(0) = g(x) = y = g(0),\ y = a_0 = g(0)$

$f^1(x) = dy/dx = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + 6a_6x^5 \ldots a_nx^{n-1}, \qquad \therefore a_1 = f'(x)$

$f^2(x) = d^2y/dx^2 = 2a_2 + 2\cdot3\,a_3x + 3\cdot4a_4x^2 + 4\cdot5a_5x^3 + 5\cdot6a_6x^4 \ldots (n-1)na_nx^{n-2} \qquad \therefore a_2 = f''(x)/2$

$f^3(x) = d^3y/dx^3 = 2\cdot3a_3 + 2\cdot3\cdot4a_4x + 3\cdot4\cdot5\,a_5x^2 + 4\cdot5\cdot6\,a_6x^3 \ldots (n-2)(n-1)na_nx^{n-3} \qquad \therefore a_3 = f'''(x)/(2\cdot3)$

: :

$f^{n-1}(x) = d^{n-1}y/dx^{n-1} = a_{n-1}(n-1)! + n!a_nx \qquad \therefore a_{n-1} = f^{n-1}(x)/(n-1)!$

$f^n(x) = d^ny/dx^n = n!a_n \qquad \therefore a_n = f^n(x)/n!$

Therefore the value of the function y = g (x) is approximated where x is close to 0 by:

$y = g(x) \approx f(0) + f^1(0)x + f^2(0)x^2/2! + f^3(0)x^3/3! + f^4(0)x^4/4! + f^5(0)x^5/5! + \ldots f^n(0)x^n/n!.$

THE TAYLOR SERIES

We now generalize the method used to acquire the Maclaurin series to any function y= g(x), where the independent variable x is near any second point in the domain a and where *x - a*< 1.

The approximating polynomial f (x) is now: where n = 1,2,3, - ∞

$y = g(x) \approx f(x) = a_0 + a_1(x-a) + a_2(x-a)^2 + a_3(x-a)^3 + a_4(x-a)^4 + a_5(x-a)^5 + \ldots + a_n(x-a)^n,$

and the approximating expansion, the Taylor series, is:

$y = g(x) \approx f(a) + f^1(a)(x-a) + f^2(a)(x-a)^2/2! + f^3(x-a)^3(a)/3! + \ldots + f^n(a)/n!\,(x-a)^n.$

APPENDIX C: CURVES IN SPACE

CURVATURE AND ITS RADIUS Ȑ.

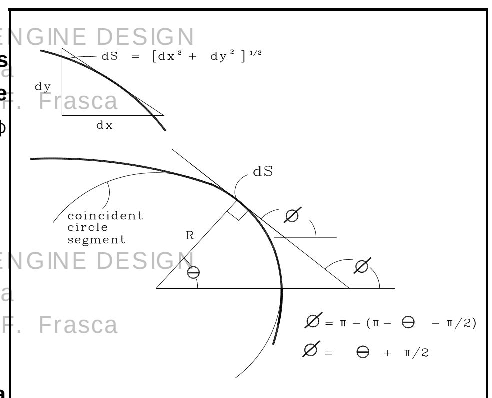
A through coverage of this topic can be seen in [36]George B. Thomas. This discussion is rudimentary. A curve's curvature, , is defined as the increment fraction dϕ/dS in the drawing. With TANϕ = dy/dx, and (TANϕ)/dϕ = SEC²ϕ,

$d\phi = TAN\phi/SEC^2\phi \equiv (dy/dx)/((dx^2+dy^2)^{1/2}/dx)^2 = (dy/dx)/(1+(dy/dx)^2)$

$Ds = [\,dy^2 + dx^2\,]^{1/2} = dx\,[\,1 + (dy/dx)^2\,]^{1/2},$

$\therefore d\phi/dS = \dfrac{[\,(dy/dx)/[\,1 + (dy/dx)^2\,]\,]}{dx\,[\,1 + (dy/dx)^2\,]^{1/2}\,]},$

$\therefore\ = d\phi/dS = (dy/dx)/(dx\,[1 + (dy/dx)^2\,]^{3/2}) \equiv (d^2y/dx^2)/[1 + (dy/dx)^2\,]^{3/2}.$



CURVATURE AND ITS RADIUS

From dS = r dθ; I. e. the curve length of an incremental distance along a curve, dS, is equal to the coincident arc increment, dθ, of a circle of radius, r, which fits, (i.e. It is coincident with all points of the topic curve increment.) and therefore dS = r dθ.

However, $\pi - (\pi - \theta - \pi/2) = \phi,\ \therefore\ d\theta = d\phi$ **and** $dS = r\,d\theta \equiv r\,d\phi.$

$Ȑ = r = 1/\ = dS/d\theta = dS/d\phi = [\,1 + (dy/dx)^2\,]^{3/2}\,]\,/\,(d^2y/dx^2).$

APPENDIX D: VISCOSITY CONSTANTS

For a comprehensive discussion of fluid viscosity see D. D. Fuller[15].

ABSOLUTE (DYNAMIC) VISCOSITY, μ_{abs}, ENGLISH SYSTEM RELATIONSHIPS:

$F = \mu_{abs} A\, dv/dh$

1 reyn = μ_{abs} = F·dh/(A·dv) = lbf·in/(in^2·in/sec) = lbf·sec/in^2

Other English system forms of absolute velocity sometimes encountered:

1 reyn = (slug·(12 in/sec^2)·sec/in^2 = 12 slug/(in-sec)
= (slug·ft/sec^2)·sec/[in·(ft/12in)]^2 = 144 slug/(ft-sec)
= [lbm·(32.174·12 in/sec^2)]·sec/in^2 = 386.088 lbm/(in-sec)
= [lbm·(32.174 ft/sec^2)·sec/[in·(ft/12 in)]^2 = 4633.056 lbm/(ft-sec)

1 microreyn(μreyn) = 1 x 10^{-6} lbf·sec/in^2
= 1 x 10^{-6} lbf·sec/[in^2·ft^2/(144 in^2)] = 144 x 10^{-6} lbf·sec/ft^2
= 12 x 10^{-6} slug/(in-sec)
= 386.04 x 10^{-6} lbm/(in-s)

ABSOLUTE (DYNAMIC) VISCOSITY, μ_{abs}, cgs (centimeter, gram, second) SYSTEM RELATIONSHIPS:

1 POISE (P) = μ_{abs} = F·dh/(A·dv) = dynes·cm·sec/(cm^2·cm) = dynes·sec/cm^2

More frequently encountered centipoise (cP): 1 cP = 1/100 P = 1 x 10^{-2} P

CONVERSION CGS TO ENGLISH:

1 P = dynes·sec/cm^2 = 1 dynes·(lbf/448,000 dynes)·sec/[cm·(in/2.54 cm)]^2
= 1.4401 x 10^{-5} reyns = 14.401 μreyn

1 cP = 10^{-2} dynes·sec/cm^2 = 1.4401 x 10^{-7} reyns = .14401 μreyn

CONVERSION OF CGS TO SI

1 dyne = gm·(kg/1000gm)·cm(m/100 cm)/sec^2 = 1x10^{-5} newton
1 P = dynes·sec/cm^2 = 1x10^{-5}N·sec/(cm·m/100cm)^2 = .1 N·sec/m^2 = .1 Pa·sec
1 cP = .001 Pa·sec

CONVERSION OF SI TO ENGLISH

1 Pa·sec = 10 P = 1000 cP = 144.01 μreyn = 144.01 x 10^{-6} reyn
1 Pa·sec = (144.01 x 10^{-6} reyn)(4633.056 lbm/ft-sec)/reyn = .667206 lbm/(ft-sec)
1 Pa·sec = (144.01 x 10^{-6} reyn)(144 lbf·sec/ft^2)/reyn = .02073744 lbf·sec/ft^2
= (144.01 x 10^{-6} reyn)(144 slug/ft-sec)/reyn = .020736 slug/(ft-sec)

ENGLISH TO SI

1 reyn = 1/1.4401 x 10^{-4}(Pa·sec) = 6943.96222 Pa·sec
1 lbm/(ft-sec) = 1.488164 Pa·sec
1 lbf·sec/ft^2 = 47.88026 Pa·sec
1 slug/(ft-sec) = 47.88026 Pa·sec

KINEMATIC VISCOSITY, ν.

$\nu = \mu/\varrho$, where ϱ = mass density, usually slugs/in^3, in the english system.
Specific weight, 1bf/ft^3 = γ = ϱ·g/ft^3 = ϱ_{slugs}·g/ft^3 = (ϱ_{lbm}·g/32.174)/ft^3
For H_20 at STP γ = 62.4 lbf/ft^3 and
ϱ_{H2O} = γ/g = (62.4 lbm·g/32.174)/ft^3 = 62.4 lbm/ft^3 = 62.4 lbm/1728 in^3 = .0361111 lbm/in^3
= 1.93945 slugs/ft^3 = .001122367 slugs/in^3

When density is given in specific gravity, sg., then:
ϱ = sg. $*$ 0.03611111 lbm/in^3 = sg. $*$ 0.001122367 slugs/in^3.

Dimensionally: $\nu = \mu/\varrho$ = reyn/(slugs/in^3) = [12 slugs/(in-sec)]/(slugs/in^3) = in^2/sec; i.e. area/sec.